CALCULATION OF REINFORCED CONCRETE MEMBERS STRENGTH BY NEW CONCEPT

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Abstract

The paper presents a generalized deformation model for the calculation of building structures made of reinforced concrete, which is considered as composite material formed of reinforcement and concrete. On this basis, it is proposed to introduce new concept – the design strength of reinforced concrete – into the modern nonlinear theory of reinforced concrete. This allows us to reduce the structural analysis of reinforced concrete members to the methodology of calculations introduced in the classical "strength of materials", which contributes to a significant simplification and acceleration of the design process of both single members and structures in general. The main advantage of the structural analysis method with the use of the reinforced concrete strength concept is that it remains unchanged when changing the stress-strain diagrams of reinforcement and concrete, the introduction of new information about these materials properties, technological parameters, loads, etc. Only one parameter – the design strength of the reinforced concrete, which is synthesized for all of the above factors, is changed in the proposed analysis method. This approach to the structural analysis of reinforced concrete members makes it universal for all deformation types of reinforced concrete members. The paper demonstrates and confirms the possibility of using the developed methodology not only in the bearing capacity calculations of reinforced concrete members, but also in calculations of the crack formation and in checking the crack width.

Keywords: Concrete, reinforcement, strength, analysis.

1. General provisions

1.1. Introduction

The first methods of flexural reinforced concrete members' strength determination were based on the method of structural analysis, which was proposed by Navier in 1826. This method was used for members of elastic materials. The first mention of reinforced concrete occurs in the middle of the eighteenth century. At the same time, the method of structural analysis for reinforced concrete members began to be used only in 1887 – 1892. The main prerequisites of the method included the following: the validity of Hooke's law for materials and the hypothesis of plane sections, the cross-section was reduced to a conditional homogeneous, the ratio of elasticity modules was assumed constant (Stoljarov 1941, Sahnovskij 1958). The essence of the method was to determine the stresses σ_i in structures caused by the action of external loads and compare them with allowable stresses $[\sigma_i]$ for the structure material on the condition: $\sigma_i \leq [\sigma_i]$. Modern methods of reinforced concrete members structural analysis (Eurocode 2 2006, MacGregor & Wight 2005, Pavlikov 2007) provide using of nonlinear stress-strain diagrams for materials subject to validity of the linearity of longitudinal strains in the section of the members. This leads to necessity of iterative methods introduction for designing structures using computational techniques. It is commonly accepted that such calculations should be verified by classical and practical methods. One of such methods, which allows performing structural analysis of reinforced concrete members based on commonly accepted expressions of the classical strength of materials (Kochkarov 2015), is proposed in this article. The purpose of this method is to simplify the structural analysis of reinforced concrete members by introducing the synthesized concept of the design reinforced concrete strength, taking into account the strength characteristics of concrete and reinforcement, the reinforcement ratio and the location of reinforcement in the cross-section.

1.2. Prerequisites of analysis

The following prerequisites for structural analysis are accepted:

1) Concrete stress-strain diagram $\sigma_c = f(\varepsilon_c)$ is used in the form of nonlinear dependencies that conformed by generally accepted deformation criteria.

2) Reinforcing steel stress-strain diagram $\sigma_s = f(\varepsilon_s)$ is described by bi-linear Prandtl diagram with corresponding values of the limiting points.

3) Hypothesis of plane cross sections is used.

4) The tensile strength of concrete is ignored.

As destruction criteria for nonlinear stress-strain diagrams of materials, the following are accepted:

1) The extremum is achieved by the function of the load-bearing capacity $dM_{Ed} / d\varepsilon_c = 0$ within ε_{c1} ... ε_{cu} with simultaneous the tensile reinforcement yield.

2) The ultimate concrete compressive strains ε_{cu} and the yield strength of the tensile reinforcement in the absence of the extremum of the load-bearing capacity are achieved in the cross-section.

3) The extremum of the load-bearing capacity function $dM_{Ed} / d\varepsilon_c = 0$ is achieved within ε_{c1} ... ε_{cu} without reaching the yield strength of the reinforcement.

4) The ultimate concrete compressive strains ε_{cu} are achieved without achieving the yield strength of the tensile reinforcement and in the absence of an extremum in the function of bearing capacity within ε_{c1} ... ε_{cu}.

5) The tensile reinforcement strains reach ε_{ud}.

The described criteria are presented by the system:

$$\begin{cases} \dfrac{dM_{Ed}}{d\varepsilon_c} = 0 \text{ when } \varepsilon_c \le \varepsilon_{cu}; \\ \sigma_{si} = \varepsilon_{si} E_s \le f_{yd}, \; i = 1...n; \\ \varepsilon_{si} \le \varepsilon_{ud}, \; i = 1...n. \end{cases} \tag{1}$$

2. Introduction of the concept of reinforced concrete strength

2.1. Deformation mode of the flexural reinforced concrete member section

For the above conditions, the deformation mode of the section of the flexural reinforced concrete member is realized in Figure 1. The equations of equilibrium are:

$$N_c = \sum_{i=1}^{n} N_{si}; \tag{2}$$

$$M_c + \sum_{i=1}^{n} M_{si} = M_{Ed}, \tag{3}$$

where N_c, N_{si} – respectively, the resultant forces in concrete compressed area and in the i-th reinforcement bar; M_c, M_{si} – respectively, the bending moments of the internal forces in the concrete and in the i-th reinforcement bar about the neutral axis; n – the number of reinforcing bars in the cross section; M_{Ed} – the bending moment of the external forces.

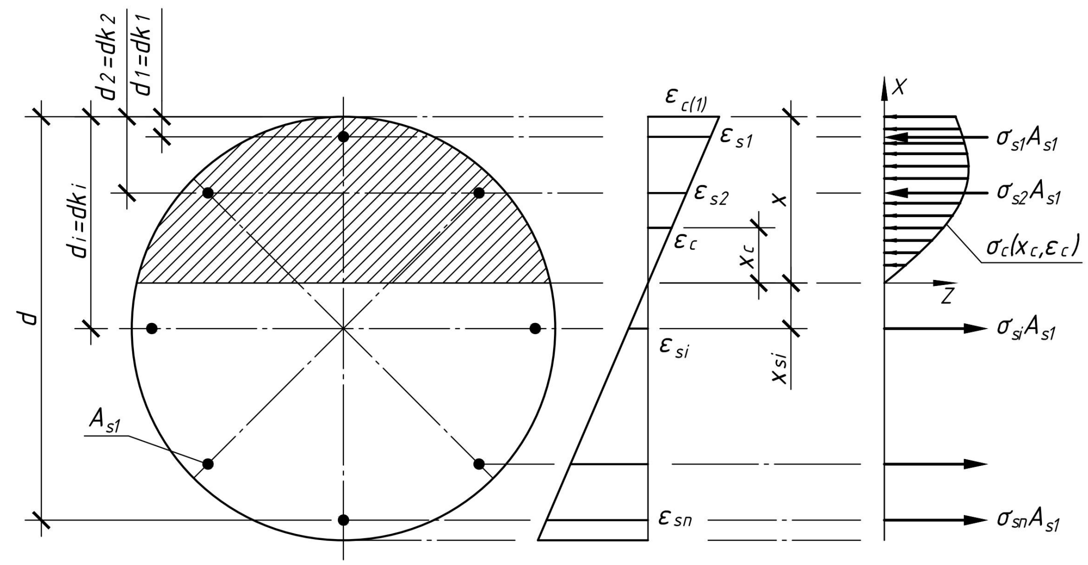

Figure 1. Deformation mode of flexural reinforced concrete member.

After simple transformations, taking advantage of the prerequisites, it is received:

$$\int_0^{A_c} \sigma_c(x_c,\varepsilon_c)dA_c = A_{s1}E_s\varepsilon_{c(1)}\sum_{i=1}^{n}\left(\frac{k_i d}{x}-1\right); \tag{4}$$

$$\int_0^{A_c} \sigma_c(x_c,\varepsilon_c)x_c dA_c + A_{s1}E_s\varepsilon_{c(1)}x\sum_{i=1}^{n}\left(\frac{k_i d}{x}-1\right)^2 = M_{Ed}, \tag{5}$$

where σ_c (x_c, ε_c) – the stress distribution function in the compressed concrete area A_c; A_{s1} – the area of one reinforcement bar in the section of the reinforced concrete member; $k_i = d_i / d$ (Fig. 1); x – the neutral axis depth; x_c – the coordinate of the application point of the resultant force in the elementary concrete compressed area dA_c.

Both parts of equation (4) are divided by the effective area A of the concrete cross section, and equation (5) is divided by the elastic resisting moment W of the effective concrete cross section.

In the case of rectangular cross sections there are

$$A = bd,\ W = bd^2/6, \tag{6}$$

for circular section

$$A = \pi d^2/4,\ W = \pi d^3/32. \tag{7}$$

2.2. Equilibrium equations for a circular cross section

Now, for example, consider a reinforced concrete member with a circular cross-section (Fig. 1). For such a member equations (4) and (5), taking into account the dependencies (7) and the designations in Figure 1, take the form:

$$\frac{4\int_0^{A_c} \sigma_c(x_c,\varepsilon_c)dA_c}{\pi d^2} = \frac{\rho_l E_s\varepsilon_{c(1)}}{n}\sum_{i=1}^{n}(k_i k-1); \tag{8}$$

$$\frac{32\int_0^{A_c} \sigma_c(x_c,\varepsilon_c)x_c dA_c}{\pi d^3} + \frac{8\rho_l E_s\varepsilon_{c(1)}}{nk}\sum_{i=1}^{n}(k_i k-1)^2 = \frac{M_{Ed}}{W}. \tag{9}$$

In deriving the dependencies (8) and (9), the following notations are introduced

$$\rho_l = \frac{4nA_{s1}}{\pi d^2},\ k = \frac{d}{x}. \tag{10}$$

2.3. The concept of reinforced concrete strength

The structural analysis method of reinforced concrete is proposed with introduction the concept of reinforced concrete strength. In the general case, it can be represented as an expression

$$f_i(a_1,...a_n) = \frac{F_{Ed}}{f(b_1,...b_n)}, \tag{11}$$

where f_i (a_1, ... a_n) – the design strength of the composite material in the cross section of the member under the condition of its destruction on the i-th material, MPa; F_{Ed} – the design value of external force factor, which corresponds to the destruction state of the member's cross section; f (b_1, ... b_n) – the corresponding geometric characteristic; a_1, ... a_n – physical-mechanical parameters of the materials; b_1, ... b_n – geometric parameters of the cross section.

Summarizing expression (11) it should be noted that the reinforced concrete strength will be determined by the minimum value of resistance, provided that the member's cross section is destroyed in all materials of which it is composed

$$f = \min\left(f_1(a_1,...a_n),...f_i(a_1,...a_n),...f_n(a_1,...a_n)\right), \tag{12}$$

where f – the total design strength of the composite material (reinforced concrete) in the section.

The left side of the expression (9) in combination with the criteria (1) is a formula for determining the strength of reinforced concrete in a circular cross section of the flexural reinforced concrete members

$$f_{zM} = \frac{32\int_0^{A_c} \sigma_c(x_c,\varepsilon_c)x_c dA_c}{\pi d^3} + \frac{8\rho_l E_s \varepsilon_{c(1)}}{nk}\sum_{i=1}^{n}(k_i k - 1)^2. \tag{13}$$

Similar expressions are obtained for reinforced concrete flexural members of a rectangular cross-section with single, double and distributed reinforcement.

3. Practical application of the developed method

3.1. General condition of flexural reinforced concrete members bearing capacity

In order to use the concept of reinforced concrete strength in calculations of reinforced concrete members bearing capacity, tables of its values are compiled depending on the reinforcement ratio, concrete and reinforcement classes (Tables 1, 2, 3). In calculating these values, the Eurocode 2 stress-strain relation for non-linear structural analysis was adopted.

The general condition of flexural reinforced concrete members bearing capacity has the form

$$f_{zM} \geq \frac{M_{Ed}}{W}, \tag{14}$$

where $f_{zM} = f$ (C, ρ_l, f_{yd}) – the design value of reinforced concrete strength in flexural members, is taken in tables 1 – 3; W – elastic moment of resistance of the effective concrete section.

Condition (14) allows solving all problems of designing reinforced concrete members: check and determine the strength of the cross-section, calculate the reinforcement area. This method permits to reduce the calculation of reinforced concrete members to the method of classical strength of materials, but taking into account nonlinear deformation of materials. The proposed method of calculation is not simplified. After all, at the values of the corresponding reinforcement ratios given in the table, the cross section strength calculated by (14) completely coincides with the strength defined by Eurocode 2 in the software complexes.

Table 1. Design strength values of reinforced concrete f_{zM1} for flexural members with single reinforcement, MPa.

Strength classes for concrete	Longitudinal reinforcement ratio ρ_l							
	0.05	0.5	1	1.25	1.75	2	2.5	3
	f_{yd} = 375 MPa (A400C)							
C8/10	1.078	9.205	13.545	13.992	14.590	14.802	15.123	15.354
C12/15	1.083	9.714	16.956	18.643	19.626	19.982	20.531	20.936
C16/20	1.086	10.033	18.233	21.646	25.196	25.736	26.582	27.217
C20/25	1.088	10.220	18.981	22.814	29.359	30.978	32.138	33.021
C25/30	1.089	10.326	19.403	23.474	30.679	33.807	36.553	37.645
C30/35	1.089	10.405	19.718	23.966	31.643	35.072	40.687	42.062
C32/40	1.090	10.465	19.961	24.345	32.386	36.043	42.630	46.307
C35/45	1.091	10.522	20.189	24.702	33.086	36.957	44.058	50.238
C40/50	1.091	10.561	20.342	24.941	33.554	37.568	45.013	51.679
C45/55	1.091	10.592	20.468	25.137	33.939	38.071	45.799	52.810
C50/60	1.092	10.623	20.590	25.328	34.313	38.560	46.563	53.911

Table 2. Design strength values of reinforced concrete f_{zM2} for flexural members with double symmetrical reinforcement, MPa.

Strength classes for concrete	Longitudinal reinforcement ratio ρ_l						
	0.05	0.5	1	1.25	1.5	2	2.5
	f_{yd} = 375 MPa (A400C)						
C8/10	1.089	10.341	20.621	25.764	30.907	41.196	51.486
C12/15	1.108	10.370	20.647	25.788	30.930	41.216	51.504
C16/20	1.128	10.405	20.679	25.818	30.959	41.243	51.529
C20/25	1.146	10.438	20.712	25.851	30.990	41.272	51.556
C25/30	1.159	10.466	20.740	25.877	31.016	41.297	51.580
C30/35	1.171	10.494	20.766	25.904	31.042	41.322	51.604
C32/40	1.182	10.520	20.794	25.930	31.068	41.347	51.628
C35/45	1.194	10.550	20.827	25.962	31.098	41.376	51.656
C40/50	1.203	10.574	20.855	25.989	31.124	41.399	51.678
C45/55	1.212	10.597	20.882	26.017	31.151	41.423	51.699
C50/60	1.221	10.623	20.915	26.050	31.184	41.453	51.726

Table 3. Design strength values of reinforced concrete f_{zMk} for flexural members with circular sections, MPa.

Strength classes for concrete	Longitudinal reinforcement ratio ρ_l						
	0.05	0.5	1	2	3	4	5
	f_{yd} = 375 MPa (A400C)						
C8/10	0.868	6.852	12.716	23.338	33.651	43.931	54.200
C12/15	0.887	7.161	13.138	24.278	34.619	44.888	55.128
C16/20	0.901	7.473	13.583	25.232	35.634	45.873	56.055
C20/25	0.909	7.643	13.963	25.738	36.480	46.678	56.787
C25/30	0.915	7.769	14.251	26.141	37.089	47.248	57.285
C30/35	0.919	7.877	14.503	26.508	37.592	47.709	57.672
C32/40	0.922	7.969	14.720	26.842	38.033	48.126	58.034
C35/45	0.925	8.061	14.932	27.184	38.420	48.486	58.332
C40/50	0.927	8.122	15.073	27.426	38.685	48.745	58.559
C45/55	0.928	8.171	15.181	27.621	38.856	48.909	58.696
C50/60	0.930	8.218	15.281	27.814	39.015	49.071	58.839

3.2. Decomposition of sections

In accordance with the described method, it is also possible to carry out calculations members of the T-section using tables for rectangular members. The effective concrete moment of resistance when passing the neutral axis in the rib of the T-section, it is necessary to take equal

$$W = \frac{b_f d^2}{6} - \frac{(b_f - b)(d - h_f)^2}{6}. \quad (15)$$

Dimensions are given in Figure 2.

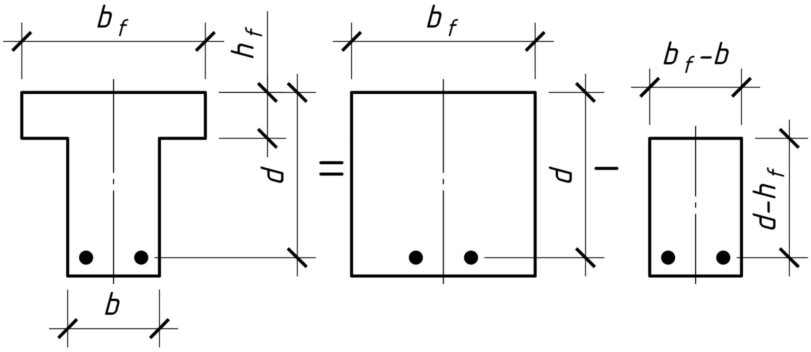

Figure 2. Transformation of the T-section.

Consider the derivation of the expression (15). For that, it is shown the possibility of decomposing a rectangular section on simple figures, provided that they have the same strength (Fig. 3).

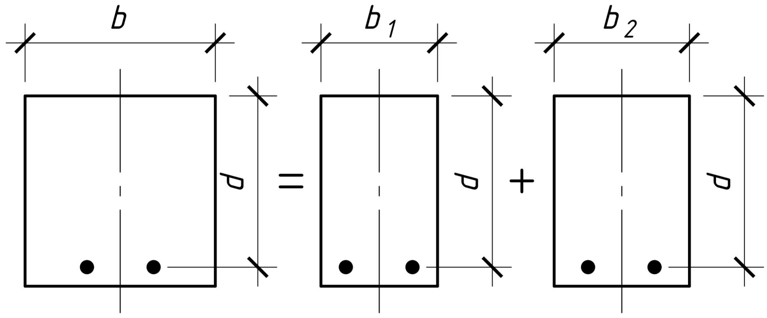

Figure 3. Decomposition of rectangular section.

The bearing capacity separately for each section is determined using formula (14):

$$M_{Ed} = f_{zM}\frac{bd^2}{6};\ M_{Ed1} = f_{zM(1)}\frac{b_1 d^2}{6};\ M_{Ed2} = f_{zM(2)}\frac{b_2 d^2}{6}. \quad (16)$$

Taking into account the equality of strength, it is written $M_{Ed} = M_{Ed1} + M_{Ed2}$ or

$$f_{zM}\frac{bd^2}{6} = f_{zM(1)}\frac{b_1 d^2}{6} + f_{zM(2)}\frac{b_2 d^2}{6}. \quad (17)$$

Equation (17) is correct in these conditions

$$b = b_1 + b_2;\ f_{zM} = f_{zM(1)} = f_{zM(2)}. \quad (18)$$

The equality of the design strength of reinforced concrete for specific sections shows the equality of the reinforcement ratio. Thus, the bearing capacity of the reinforced concrete member of a rectangular cross section can be determined, knowing the bearing capacity of each of the simple figures in which it is broken for the same reinforcement ratio and the effective depth.

Similarly, calculations of the T-section members can be performed. Because in the calculation of T-section members it is not known in advance, where the neutral axis is located (in the flange or in the rib) each section must be calculated twice. The first calculation is for the member of the T-section with the moment of resistance W by (15) and the second calculation is for the member of the rectangular cross section $b_f \times d$ with the moment of resistance $b_f d^2 / 6$. From the two received values, in determining the area of reinforcement, the lager value should be taken and, the lesser value should be used in determining the bearing capacity.

3.3. Calculation example

The application of the proposed calculation method is considered by the example 40 given in (Pysarenko 2004) with additional conditions. According to the example, the beam (Fig. 4) has a load of 100 kN, evenly distributed over the span. Material of the beam is C20/25 concrete, reinforcement A400C. It is necessary to get different variants of the cross sections of the beam, which are shown in Figure 4, with the reinforcement ratio equal to 1%.

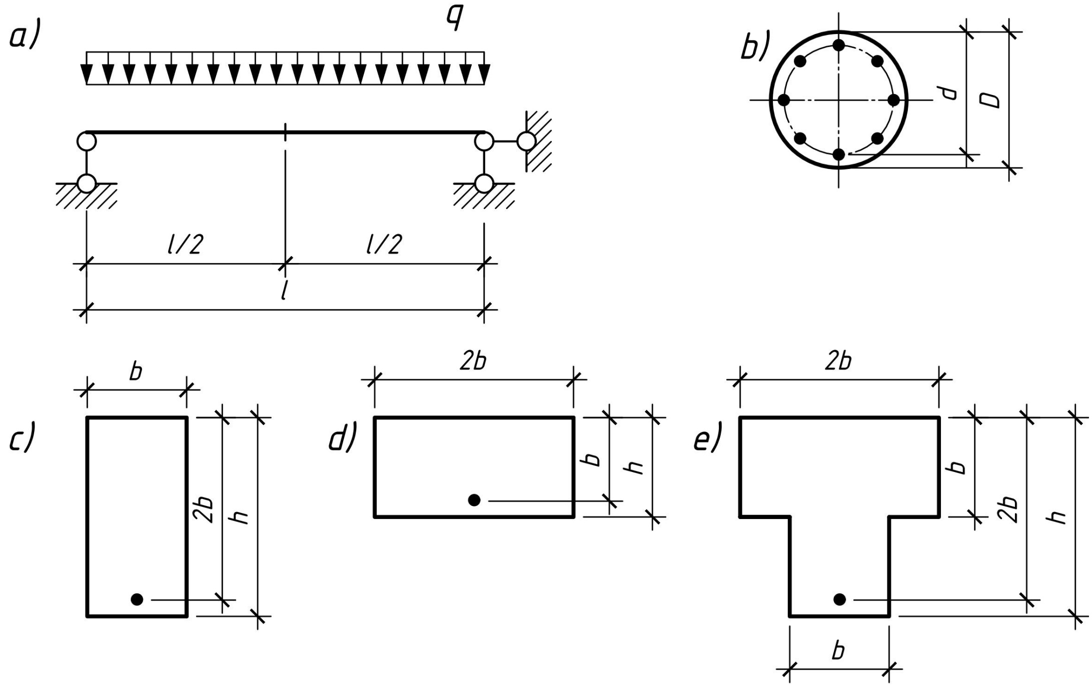

Figure 4. To the solution of the problem: *a*) design scheme; *b*), *c*), *d*), *e*) variants of the cross section.

1. The value of the maximum bending moment for the beam (taken from Pysarenko 2004)

$$M_{Ed,\max} = q\frac{l^2}{8} = \frac{ql \times l}{8} = \frac{P \times l}{8} = \frac{100 \times 1.6}{8} = 20\,\text{kNm}.$$

2. From Table 1 and Table 3 the design strength of reinforced concrete is determined:

– for section *b*) f_{zMk} = 13.96 MPa at ρ_l = 1%;

– for sections *c*) and *d*) f_{zM1} = 18.98 MPa at ρ_l = 1%;

– for section *e*) – for T-section f_{zM1} = 18.98 MPa at $\rho_{l(1)}$ = 1%; – for corresponding rectangular section the reinforcement ratio $\rho_{l(2)}$ is determined with relation $\rho_{l(2)} = \frac{3}{4}\rho_{l(1)} = \frac{3}{4}1\% = 0.75\%$, where $\rho_{l(1)} = \frac{A_s}{3b^2} \times 100\%$; $\rho_{l(2)} = \frac{A_s}{4b^2} \times 100\%$. From Table 1 at $\rho_{l(2)}$ = 0.75% for corresponding rectangular section it is found f_{zM1} = 14.60 MPa.

3. The effective moment of resistance for the proposed variants of the concrete cross sections (Fig. 4):

– for section *b*) $W = \pi d^3/32$;

– for section *c*) $W = 4b^3/6$;

– for section *d*) $W = b^3/3$;

– for section *e*) – for T-section $W_1 = 7b^3/6$; – for corresponding rectangular $W_2 = 4b^3/3$.

4. For the cross section *e*), the position of the neutral axis should be found. For this the values of the bearing capacity for the T-section (M_1) and corresponding rectangular (M_2) are written

$$M_1 = W_1 f_{zM1} = 7b^3/6\times18.98 = 22.14b^3\,;\; M_2 = W_2 f_{zM1} = 4b^3/3\times14.60 = 19.47b^3.$$

Since $M_2 < M_1$, the neutral axis is in the flange. The calculation is made for a rectangular section with the corresponding dimensions.

5. Required moments of resistance from the expression (14)

– for section *b*) $W = \dfrac{M_{Ed,\max}}{f_{zMk}} = \dfrac{20\times10^3}{13.96} = 1432.7\,\text{cm}^3$;

– for sections *c*) and *d*) $W = \dfrac{M_{Ed,\max}}{f_{zM1}} = \dfrac{20\times10^3}{18.98} = 1053.7\,\text{cm}^3$;

– for section *e*) $W = \dfrac{M_{Ed,\max}}{f_{zM1}} = \dfrac{20\times10^3}{14.60} = 1369.9\,\text{cm}^3$.

6. Required sizes for the proposed cross-sections:

– for section *b*) $d = \sqrt[3]{32W/\pi} = \sqrt[3]{32\times1432.7/3.14} = 24.4\,\text{cm}$;

– for section *c*) $b = \sqrt[3]{6W/4} = \sqrt[3]{6\times1053.7/4} = 11.7\,\text{cm}$;

– for section *d*) $b = \sqrt[3]{3W} = \sqrt[3]{3\times1053.7} = 14.7\,\text{cm}$;

– for section *e*) $b = \sqrt[3]{3W/4} = \sqrt[3]{3\times1369.9/4} = 10.0\,\text{cm}$.

Determination of reinforcement should be done using respective reinforcement ratio.

4. Conclusions

1) The strength characteristic of reinforced concrete is synthesized, which using substantially simplifies the practical method of structural analysis of flexural reinforced concrete members based on the deformation model.

2) Using the developed method, the tables have been constructed for easily and quickly performing the calculation of the bearing capacity of reinforced concrete beams according to the non-linear analysis method without the use of computer programs.

3) Similar tables are developed for the calculation of reinforced concrete members in other deformation modes.

4) In order to simplify the work of designers, engineers and students of Civil Engineering specialties it is expedient to include tables into the reinforced concrete members design rules.

5) The proposed method gives the opportunity to teach it in the Strength of Materials base course for the calculation of reinforced concrete members.

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