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# *DISCRETE MODELING OF MESH FRAMES OF COVERING SURFACES BY CHAINS OF SUPERPOSITIONS*

In this article we have researched of the construction of a chain of successive superpositions of two pairs of points for modeling a frame of a curved surface. It is shown that a superposition of n points of a discrete analogue of any given surface can be replaced by a chain of successive superpositions, considering a value of recurrent dependence.

**Keywords:** discrete geometric modeling, covering surface, static-geometric method, geometric apparatus of superpositions, value of recurrent dependence, coefficients of superposition.

Modern construction often uses covers in the form of shells of complex geometric shapes, which allows creating unique architectural structures, owing to a wide variety of forms. Calculations of such buildings, considering physical properties of materials, features of manufacturing and installation technology, require the presentation of an object information in the discrete form. Because of this it is expedient to carry out the process of their formation in the discrete form right away.

A static-geometric method of discrete geometric modeling of curves and surfaces [1] allows to get discrete frames of curved surfaces under an external forming load and, besides this, it is simple and obvious. Usage of the static-geometric method provides discrete frames of curved surfaces on an arbitrary bearing contour.

The mathematical apparatus of the static-geometric method is based on solving cumbersome systems of linear equations, which complicates the process of computer calculations.

In [2, 3], the authors of that article have shown some approaches to the definition of discrete analogs of certain functional dependencies, using the geometric apparatus superpositions for one-dimensional point sets. It allows creating discrete contours without making and solving cumbersome systems of equations. Thesis [4] was devoted to studying some properties of superpositions of discrete point sets. In particularly, the author has considered a possibility of a management of the shapes of a stretched mesh on the basis of a functional addition. Authors of [5, 6] studied the same problems. In [7] it was shown that a superposition of n points can be replaced by a chain of successive superpositions.

Usage of the geometric apparatus of superpositions together with the static-geometric method allows increasing efficiency significantly and enhancing an ability of the process of digital modeling of surfaces.

The aim of this article is to study possible variants of the organization of chains of successive superpositions of four points for discrete modeling of mesh frames of curved surfaces.

Since one of the basic principles of static-geometric method of modeling of curved surfaces is the management of the shapes of surfaces by changing a type of distribution of an external forming load, it is advisable to consider the value of this load, forming discrete frames of surfaces by chains of successive superpositions.

Since concentrated efforts (a load value) at the nodal points include the existence of balancing efforts in network connections, then creating digital images, using the geometric apparatus of superpositions, we shall use the term "value of recurrent dependence" instead of the term "value of an external load". Obviously, both of these terms are identical.

Formula

$$z_{i,j} = k_1 z_{i-1,j} + k_2 z_{i+1,j} + k_3 z_{i,j-1} + k_4 z_{i,j+1} \text{ ,}$$

where $k_1$ , $k_2$ , $k_3$ , $k_4$ are coefficients of superposition of the coordinates of z given adjacent grid points,
is identical to finite-difference four-points dependence

$$4z_{i,j} = z_{i-1,j} + z_{i+1,j} + z_{i,j-1} + z_{i,j+1} \text{ .}$$

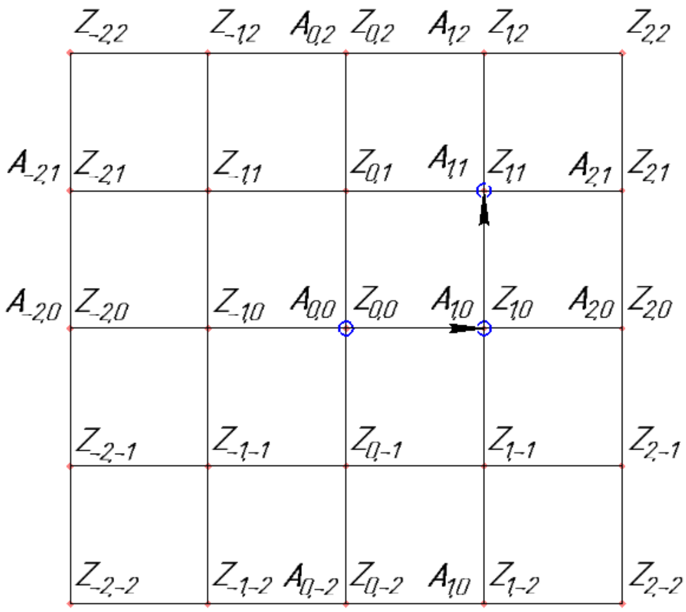
Therefore, for forming a discrete frame of a surface, based on a superpositions of four given adjacent grid points, the value of recurrent dependence, which is the prototype of an external forming load, can be written as:

$$P_i = z_{i,j} - k_1 z_{i-1,j} - k_2 z_{i+1,j} - k_3 z_{i,j-1} - k_4 z_{i,j+1} \text{ ,}$$

where $P_i$ is the discrete value of recurrent dependence.

Providing that $k_1+k_2+k_3+k_4=1$ ,

$$P_i=z_{i,j}-k_1z_{i-1,j}-k_2z_{i+1,j}-k_3z_{i,j-1}-\left(1-k_1-k_2-k_3\right)z_{i,j+1} .$$




Picture1. Plan of the discrete point frame of a part of the surface

A superposition of n contour points of the discrete mesh frame of a surface can be replaced by a chain of successive superpositions of corresponding pairs of points.

A plan of the discrete point frame of a certain surface is shown on Picture 1. Superposition $A_{0,0}$ of two pairs of contour grid points $A_{-2,0}$ , $A_{2,0}$ and $A_{0,-2}$ , $A_{0,2}$ of this surface can be written as:

$$u_{A_{0,0}}=u_{A_{-2,0}}\cdot k_1^{(-2,0)-(2,0)-(0,-2)-(0,2)}+u_{A_{2,0}}\cdot k_2^{(-2,0)-(2,0)-(0,-2)-(0,2)}+ u_{A_{0,-2}}\cdot k_3^{(-2,0)-(2,0)-(0,-2)-(0,2)}+u_{A_{0,2}}\cdot\left(1-k_1^{(-2,0)-(2,0)-(0,-2)-(0,2)}- -k_2^{(-2,0)-(2,0)-(0,-2)-(0,2)}-k_3^{(-2,0)-(2,0)-(0,-2)-(0,2)}\right)+P_{0,0} , \quad (1)$$

where $u_{A_{0,0}}$ is a generalized coordinate of the point, which was gotten as a result of superposition of given contour grid points $A_{-2,0}$ , $A_{2,0}$ и $A_{0,-2}$ , $A_{0,2}$ ;

$k_1^{(-2,0)-(2,0)-(0,-2)-(0,2)}$ , $k_2^{(-2,0)-(2,0)-(0,-2)-(0,2)}$ $k_3^{(-2,0)-(2,0)-(0,-2)-(0,2)}$ are coefficients of superposition of the given contour grid points;

$P_{0,0}$ is the value of recurrent dependence, which is an analog of a discrete value of external forming load, in grid point $A_{0,0}$ .

Considering the superposition of six points $A_{-2,0}$ , $A_{2,0}$ , $A_{0,-2}$ , $A_{0,2}$ , $A_{1,-2}$ , $A_{1,2}$ of a given bearing contour:

$$z_{1,0} = k_1 z_{-2,0} + k_2 z_{2,0} + k_3 z_{0,-2} + k_4 z_{0,2} + k_5 z_{1,-2} + \\ + \left(1 - k_1 - k_2 - k_3 - k_4 - k_5\right) z_{1,2} . \quad (2)$$

At a certain dependence between the coefficients of superposition (2) can be obtained as a sequence of two superpositions:

$$z_{0,0} = k_1^{1*} z_{-2,0} + k_2^{1*} z_{2,0} + k_3^{1*} z_{0,-2} + \left(1 - k_1^{1*} - k_2^{1*} - k_3^{1*}\right) z_{0,2} + P_{0,0} , \quad (3)$$

$$z_{1,0} = k_1^{2*} z_{0,0} + k_2^{2*} z_{2,0} + k_3^{2*} z_{1,-2} + \left(1 - k_1^{2*} - k_2^{2*} - k_3^{2*}\right) z_{1,2} + P_{1,0} , \quad (4)$$

where $k_i^{1*} = k_i^{(-2,0)-(2,0)-(0,-2)-(0,2)}$ , $k_i^{2*} = k_i^{(0,0)-(2,0)-(1,-2)-(1,2)}$ .

Substituting expression (3) into (4), we get

$$z_{1,0} = k_1^{2*} k_1^{1*} z_{-2,0} + \left(k_1^{2*} k_2^{1*} + k_2^{2*}\right) z_{2,0} + k_1^{2*} k_3^{1*} z_{0,-2} + \\ + k_1^{2*} \left(1 - k_1^{1*} - k_2^{1*} - k_3^{1*}\right) z_{0,2} + k_3^{2*} z_{1,-2} + \left(1 - k_1^{1*} - k_2^{1*} - k_3^{1*}\right) z_{1,2} + P_{1,0} \quad (5)$$

The superposition of eight points $A_{-2,0}$ , $A_{2,0}$ , $A_{0,-2}$ , $A_{0,2}$ , $A_{1,-2}$ , $A_{1,2}$ , $A_{-2,1}$ , $A_{2,1}$ of a given bearing contour

$$z_{1,1} = k_1 z_{-2,0} + k_2 z_{2,0} + k_3 z_{0,-2} + k_4 z_{0,2} + k_5 z_{1,-2} + k_6 z_{1,2} + \\ + k_7 z_{-2,1} + \left(1 - k_1 - k_2 - k_3 - k_4 - k_5 - k_6 - k_7\right) z_{2,1} , \quad (6)$$

can be obtained as a chain of successive superpositions (3), (4) and (7):

$$z_{1,1} = k_1^{3*} z_{1,0} + k_2^{3*} z_{1,2} + k_3^{3*} z_{-2,1} + \left(1 - k_1^{3*} - k_2^{3*} - k_3^{3*}\right) z_{2,1} + P_{1,1} , \quad (7)$$

where $k_i^{3*}=k_i^{(1,0)\text{-}(1,2)\text{-}(-2,1)\text{-}(2,1)}$ .

Substituting expression (3) into (4) and (4) into (7), we get:

$$z_{1,1}=k_1^{3*}k_1^{2*}k_1^{1*}z_{-2,0}+k_1^{3*}\left(k_1^{2*}k_2^{1*}+k_2^{2*}\right)z_{2,0}+k_1^{3*}k_1^{2*}k_3^{1*}z_{0,-2}+$$
$$+k_1^{3*}k_1^{2*}\left(1-\sum_{i=1}^{3}k_i^{1*}\right)z_{0,2}+k_1^{3*}k_3^{2*}z_{1,-2}+\left(1-\sum_{i=1}^{3}k_i^{3*}\right)z_{2,1}+ \quad (8)$$
$$+\left[k_2^{3*}+k_1^{3*}\left(1-\sum_{i=1}^{3}k_i^{2*}\right)\right]z_{1,2}+k_3^{3*}z_{-2,1}+\left(k_1^{3*}P_{1,0}+P_{1,1}\right)$$

In the article it is shown that a superposition of n points of the discrete frame of a surface can be replaced by a chain of successive superpositions, considering a value of recurrent dependence. Further investigation of possible variants of organization of chains of superpositions will allow finding analytical dependences to determinate coordinates of arbitrary nodes of discrete two-dimensional geometric images as superpositions of the coordinates of given nodes.

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