

Iryna Taras

**DESCRIPTIVE GEOMETRY,
ENGINEERING AND COMPUTER
GRAPHICS**

**Part I
Descriptive Geometry**

Lecture notes

МІНІСТЕРСТВО ОСВІТИ І НАУКИ УКРАЇНИ

Івано-Франківський національний технічний університет нафти і газу

Кафедра інженерної та комп'ютерної графіки

І. П. Тарас

**НАРИСНА ГЕОМЕТРІЯ, ІНЖЕНЕРНА
ТА КОМП'ЮТЕРНА ГРАФІКА**

**Частина I
Нарисна геометрія
КОНСПЕКТ ЛЕКЦІЙ**

**Івано-Франківськ
2013**

УДК 514.18
ББК 30.11
Т 19

Рецензент:

Шкіца Л. Є. докт. техн. наук, завідувач кафедри інженерної та комп'ютерної графіки

*Рекомендовано методичною радою університету
(протокол № 2 від 21.11. 2013р.)*

Тарас І. П.

Т 19 Нарисна геометрія, інженерна та комп'ютерна графіка. Частина 1. Нарисна геометрія: конспект лекцій – Івано-Франківськ: ІФНТУНГ, 2013. - 81 с.

МВ 02070855-7003- 2013

Конспект лекцій складено відповідно до робочої програми з дисципліни «Нарисна геометрія, інженерна та комп'ютерна графіка». Призначений для студентів напряму підготовки – 6.050304 - Нафтогазова справа.

Містить теоретичні відомості про методи проєціювання, проектування елементарних геометричних фігур, методи рішення позиційних та метричних задач, способи перетворення проєкцій, поверхні та їх взаємний перетин.

Призначений для студентів-іноземців напряму підготовки 6.050304 - Нафтогазова справа, що навчаються англійською мовою.

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INTRODUCTION

The educational discipline “Descriptive Geometry, Engineering and Computer Graphic” is complex and consists of two modules: M1 is Descriptive Geometry (first term); M2 is the Engineering and Computer Graphic (second term). Two modules are the organic whole, where one part develops and complements the other one.

Descriptive Geometry is grammar of engineering graphic and studies theoretical bases of geometrical design of three-dimensional objects by the method of projection images. Subject of a Descriptive geometry is spatial forms and their relations. A Descriptive geometry equips a student with the method of projections, which is basis of other divisions of graphic cycle and apply practically all technical disciplines. Mostly this method is used by theoretical mechanics, theory of mechanisms and machines, higher mathematics, resistance of materials, detail of machines.

As a result of learning the module ”Descriptive Geometry” a student must:

- ◆ to know theoretical bases of construction of images of points, lines, planes, separate types of the crooked lines and surfaces;
- ◆ able to decide tasks on mutual belonging and mutual crossing of geometrical figures, also on determination of natural size of separate geometrical figures;

Descriptive Geometry contains three content modules, in particular:

CM1 is the Method of projections. Projections of elementary geometrical figures;

CM2 is Position and metrical problems. Conversion of projection;

CM3 is the Surfaces and their mutual intersection.

CONVENTIONAL DENOTATION

Points of space are represented by capital letters: A, B, C, \dots

A line determined by two point A, B , is determined the line AB .

Unlimited lines of space are represented by small letters: a, b, k, \dots

Unlimited planes of space are represented by capital Greek letters: Δ, Γ, \dots

A plane determined by three points A, B, C , is called the plane ABC .

A plane determined by two intersecting lines a and b is called plane ab .

In general, given elements are denoted by early letters of the alphabet, required elements by late figures; given $A_1, A_2, A_3, B_1, B_2, B_3, C_1, C_2, C_3$, required P_1, P_2, P_3 .

The projection of a point on a plane is represented by a letter for the point and a subscript for the plane. Thus, A_1 is the projection of point A on the horizontal plane of projection Π_1 . The projection of a line is represented in the same manner. Thus, k_2 is the frontal projection or front view of the unlimited line k . A_1B_1 is the top view or horizontal projection of the line segment AB .

In tutorial guidelines are accepted such denotation.

Points are denoted by upper-case Latin letters from A to O (except of F and H) or by Arabic numerals.

Lines are denoted by down-case Latin letters.

Object	Notation	Examples
Point	upper-case Latin letters	$A, B, \dots P$
Construction point	Arabic numerals	$1, 2, \dots 10$
Line (straight line and curves)	lower-case Latin letters	l, g
Plane and surfaces	upper-case Greek letter	Δ, Γ
Projection planes	upper-case Greek letter Π with subscripts	Π_1 for the horizontal plane Π_2 for the frontal plane Π_3 for the side plane Π_4 for an additional plane
Projected object	subscript	P_1 horizontal projection of P P_2 frontal projection of P P_3 side projection of P P_4 projection on Π_4 similarly for lines
Angles	lower-case Greek letter	$\alpha, \beta, \gamma, \dots$
Lines connecting two projections of the same point, construction lines	thin line	
Line of object	continuous line	

MATHEMATICAL SIGNS

Sign	Meaning	Sample	Usage
\subset	Incidence	$A \subset a$	Point A belonged to line a
\supset	Including	$a \supset A$	Line a includes point A
\cup	Connection	$AB = A \cup B$	Line AB is a result of connection of points A and B .
\cap	Intersection	$b \cap c$	Line b intersects with line c
	Parallelism		Line m is parallel to line n
\perp	Perpendicular	$l \perp \Sigma$	Line l is perpendicular to plane Σ
\times	Crossing	$A \times b$	Line a is crossing with line b
$=$	Result	$K = c \cap d$	Point K is a result of intersection of lines c and d
\equiv	Coincidence	$A_1 \equiv B_1$	Projection of point A coincides with projection of point B
\sphericalangle	Angle	$\sphericalangle ABC$	Angle ABC
\parallel	Distance	$/A, a/$	Distance from point A to line a
\rightarrow	Go to other position	$\Pi_1 \rightarrow \Pi_2$	Go from plane Π_1 plane Π_2
	Construct	$K = m \cap \Sigma$	Construct K the point of intersection of line m with plane Σ

1 DESCRIPTIVE GEOMETRY

1.1 THE SUBJECT OF DESCRIPTIVE GEOMETRY

We live in a three-dimensional world and we often want to represent this world in a drawing, a painting or a photography. But, drawings, paintings and photos are two-dimensional.

The aim of Descriptive Geometry is to describe the three-dimensional objects by two-dimensional drawings so as to allow to reconstitute their original forms and their real dimensions.

So, **Descriptive Geometry is a method to study 3D geometry through 2D images thus offering insight into structure and metrical properties of spatial objects, processes and principles.**

1.2 TYPES OF PROJECTION

When representing a three-dimensional object on the two-dimensional surface of our retina, of a camera film, of a paper sheet, or of a TV or computer screen, the number of dimensions is reduced from three to two. The general process of reducing the number of dimensions of a given object is called **projection**. The type of projection that produces the image in our eye, on the array of electronic sensors of a digital camera is called **central projection** (Fig.1.1)

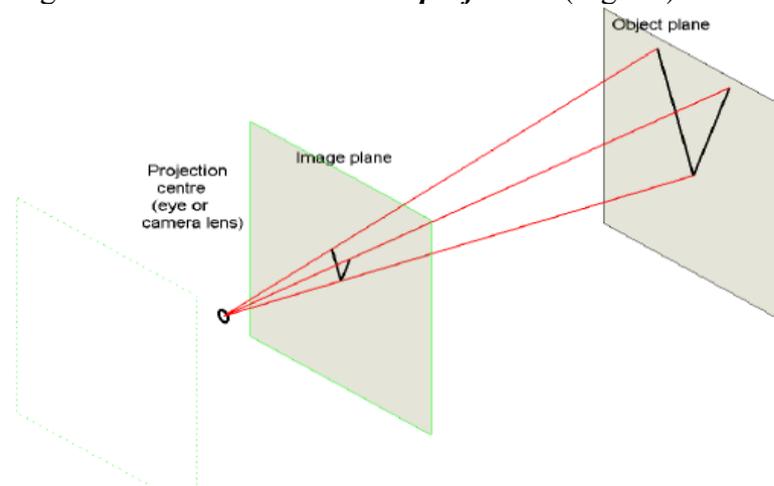


Figure 1.1

The object of projection is point B .

The projection center is point S .

The image (projection) plane is the plane Σ .

The ray that passes through **the projection center** (point S) and **the object of projection** (point B) is called **ray of projection** (t).

The image (point B_{Σ}) is the point of intersection of **the ray of projection** (t) and **the image plane** (Σ).

The image is the projection of the **object** on the **image plane** (Fig. 1.2)

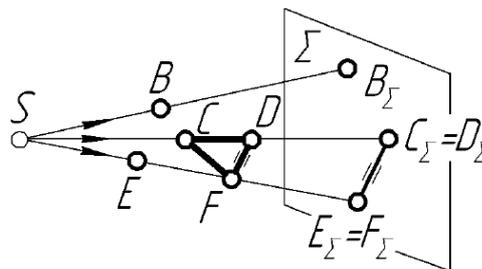


Figure 1.2

The images produced by central projection convey the sensation of depth, or, in other words, of the actual space. This is the only projection that produces the real perspective. These matters are taught in courses on **Perspective**, such as given in the University of Architecture.

Descriptive Geometry is based on another type of projection, specifically **parallel projection**, in most cases parallel, **orthogonal projection**. The result is realistic for small objects placed at long distances from the viewer. Even if the result is not realistic for other objects, the processes of projection and of measuring in the resulting drawing are simplified to such an extent that the method imposed itself in Engineering.

We make now a new, important assumption: **the projection center is sent to infinity**. Obviously, all projections rays become parallel. What we obtain is called **parallel projection** (Fig. 1.3).

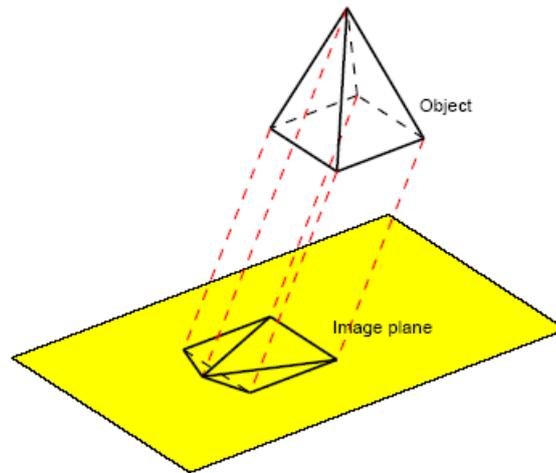


Figure 1. 3

The object of projection is point A .

The projection direction is the vector n .

The image (projection) plane is plane Σ .

The ray passing in parallel with **the projection direction** (the vector n) and **the object of projection** (point A) is called **ray of projection** (t).

The image (point A_{Σ}) is the point of intersection of **the ray of projection** (t) and **the image plane** (Σ) (Fig.1.4).

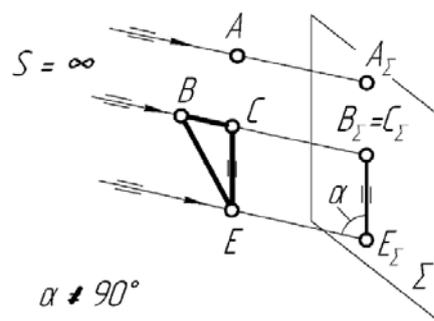


Figure 1.4

1.3 PROPERTIES OF PARALLEL PROJECTION

- The image of point is point.
- The image of straight line is straight line (in general).
- The simple ratio of three collinear points is an invariant of parallel projection.
- Parallel lines are projected as parallel.
- When the object plane is parallel to the image plane, the lengths of segments and the sizes of angles are invariant.

The first and second properties are common for both of types of projection.

In Descriptive Geometry, however, we go one step farther and we assume that *the projection rays are perpendicular to the image plane*. One simple example is shown in Figure 1.5.

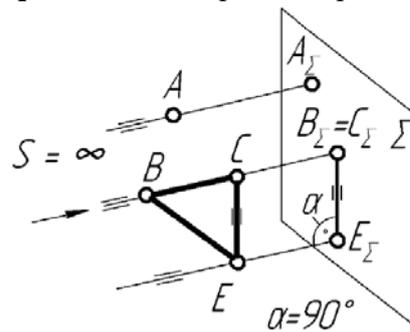


Figure 1.5

Thus we obtain the orthogonal projection. This is a particular case of the parallel projection; therefore, it inherits all the properties of the parallel projection. In addition, the assumption of projection rays perpendicular to the image plane yields a new property as the projection of a right angle.

But the projection on the one image plane is ambiguous identification of the spatial object. Projection of point *C* coincides with projection of point *D* on Fig. 1.2 and projection of point *B* coincides with projection of point *C* on Fig. 1.4 and 1.5.

1.4 THE TWO-SHEET METHOD OF MONGE

The basic idea of Descriptive Geometry is to use *two projection planes* (Fig.1.6).

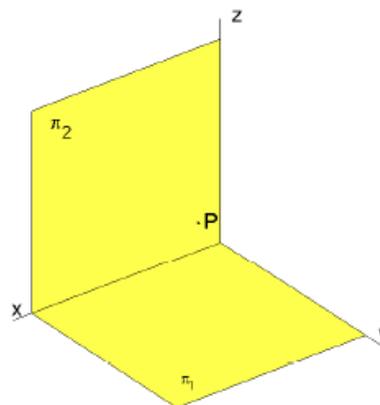


Figure 1.6

However, the first to organize the methods for doing so in a coherent system was the French mathematician Gaspard Monge (1746-1818). Therefore, he who is considered to be the founder of Descriptive Geometry. Monge was one of the scientists who accompanied Bonaparte in the Egypt campaign and was one of the founders of the École Polytechnique in Paris.

Figure 1.7 shows a point *A* and a system of Cartesian coordinates.

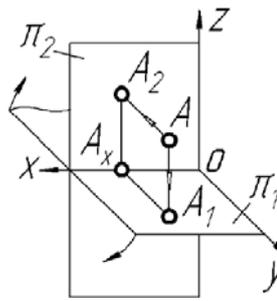


Figure 1.7

So the basic idea of Descriptive Geometry is to use two **perpendicular projection planes**, the **horizontal** one, defined by the axes x and y ; and the **frontal** one defined by the axes x and z : Let us mark the horizontal plane by Π_1 ; and the frontal one by Π_2 : These projection planes are shown in Figure 1.6. The next step is to project orthogonally the point A on Π_1 and Π_2 : The horizontal projection, A_1 is the point in which a ray emitted from A , perpendicularly to Π_1 , pierces the plane Π_1 : Similarly, the frontal projection, A_2 ; is the point in which a ray emitted from A , perpendicularly to Π_2 crosses the plane Π_2 : These projections are shown in Figure 1.7. Finally, as shown in Figure 1.7, we rotate the plane Π_2 until it is coplanar with Π_1 : We obtain thus what we call the sketch of the point A : It is shown in Figure 1.8. This figure shows that the two projections contain all the information about all the coordinates of a given point. We show in Figure 8 the sketch of the point A introduced in Figure 1.7.

The coordinates of the point A in the Monge sketch are represented on Figure 1.8.

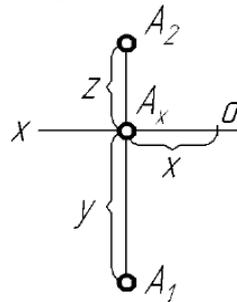


Figure 1.8

All steps of orthogonal projecting of the point A on Π_1 and Π_2 : and making of sketch are represented on Fig. 1.9.

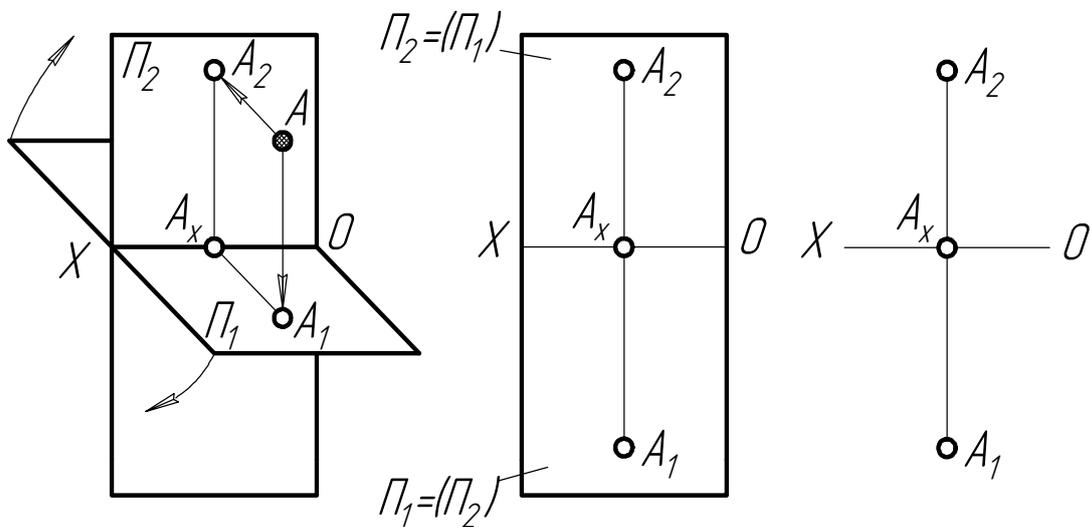


Figure 1.9

1.5 QUADRANTS OF SPACE

System of projection planes Π_1 and Π_2 divides space into four quadrants (Fig. 1.10). They differ with signs on co-ordinates of points (Table 1.1).

Table 1.1 – Signs on co-ordinates of points into different quadrants

quadrants	coordinates		
	X	Y	Z
I	+	+	+
II	+	-	+
III	+	-	-
IV	+	+	-

We use the 1st angle projection (1st quadrant), as usual in most European countries.

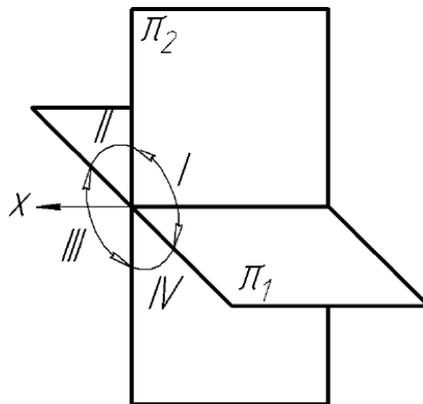


Figure 1.10

2 POINT INTO SYSTEM OF 3 COORDINATE PLANES

The two conventional planes mentioned in the preceding lecture are called planes of projection: the *horizontal* one, defined by the axes x and y ; and the *frontal* one defined by the axes x and z . But pair of axes y and z defines another plane of projection which is perpendicular to both the *horizontal* and *frontal* planes. It is called *profile* plane of projection Π_3 . In Fig. 2.1 these three planes are shown in their relative positions.

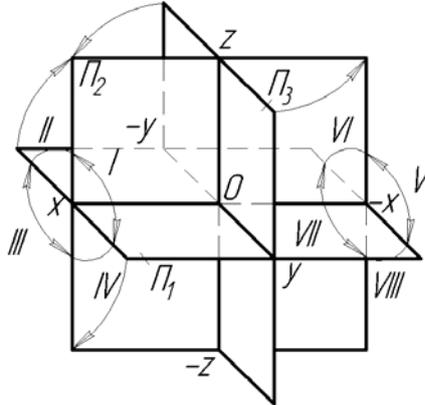


Figure 2.1

2.1 OCTANTS OF SPACE

Three planes of projection divide space into eight parts octants (Fig. 2.1), They are called octants or angles. Octants differ with signs on coordinates of points (Table 2.1).

Table 2.1 – Signs on coordinates of points into different octants

octants	coordinates			octants	coordinates		
	X	Y	Z		X	Y	Z
I	+	+	+	V	-	+	+
II	+	-	+	VI	-	-	+
III	+	-	-	VII	-	-	-
IV	+	+	-	VIII	-	+	-

The system of mutually perpendicular planes of projections constructed by us with projections of a point to them is *reversible*, allowing to define position of a point A in space, but is not the drawing (Fig. 2.2).

Straight lines AA_1 , AA_2 , AA_3 are name projecting straight lines.

For reception of the **flat complex drawing** we will transform the spatial image having combined planes of projections.

Probably the question has already entered our minds of how to represent these three coordinate planes of projection on single sheet of paper. The answer to this question is that two of the planes are revolved about certain axes until they coincide with Π_2 . Using x as an axis of revolution, part of Π_1 in front of Π_2 is revolved down, the part behind, up, of course, until Π_1 coincides with Π_2 . Using z as an axis the portion of the profile plane in front of Π_2 is revolved to the right, the portion behind, to the left, until Π_3 coincides with Π_2 (Fig. 2.1).

In the revolution just explained, the Π_2 projections of points are not affected; i.e., the Π_2 projections of points in the 1st, 2nd, 5th and 6th angles are about x (+X), and below x for the 3rd, 4th, 7th and 8th (-X), regardless of any revolution of planes; however, the Π_1 projections of all points in front of Π_2 , either in the 1st, 4th, 5th or 8th angles, being in the portion of which was revolved

down, now come below $x (+Y)$. The Π_1 projections of all points behind of Π_2 , in either the 2nd, 3rd, 6th or 7th angles ($-Y$), are found above x since that part of Π_1 was revolved up.

The Π_3 projections of points in the 1st, 4th, 5th and 8th angles are on the right of $z (+Y)$, and on the left of z for the 2nd, 3rd, 6th and 7th ($-Y$), regardless of any revolution of planes.

As the Π_1 plane is revolved about x , as an axis, the Π_1 projections of a point moves in a plane which is perpendicular to x , hence after the revolution the Π_1 projection is found on the perpendicular to x , dropped from the Π_2 projection of the point. The Π_3 projection of a point moves in a plane which is perpendicular to z , hence after the revolution the Π_3 projection is found on the perpendicular to z , because the Π_3 plane is revolved about z , as an axis (Fig.2.2).

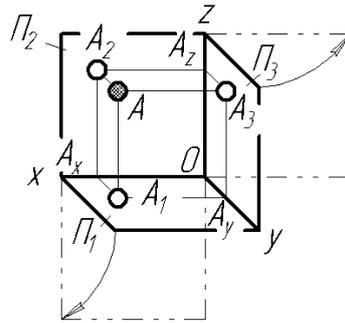


Figure 2.2

The two projections of a point must be on the same perpendicular to the axis: Π_1 and Π_2 projections must be on the perpendicular to x and Π_3 and Π_2 projections must be on the perpendicular to z (Fig. 2.3).

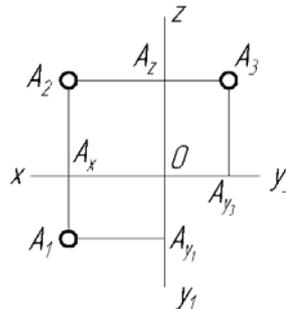


Figure 2.3

We will name this Figure 2.3 the **complex drawing**.

Thus the **horizontal** projection of point is defined by its X and Y coordinates, the **frontal** projection of point is defined by its X and Z coordinates, the **profile** projection of point is defined by its Z and Y coordinates. We know that the two projections of a point contain all the information about all its coordinates (Fig.1.8). Hence any third projection of point is not independent and may be constructed based on the two others (Fig. 2.4).

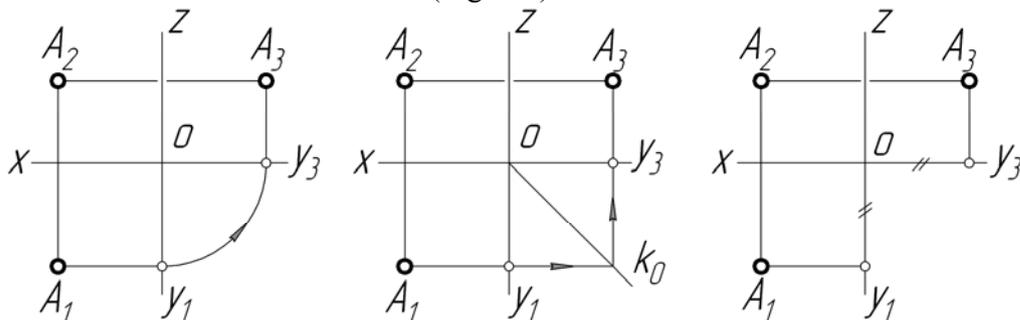


Figure 2.4

The complex drawing of points in the 1st, 2nd, 3rd, 4th, 5th, 6th, 7th and 8th angles are represented on corresponding Fig. 2.5 – 2.13.

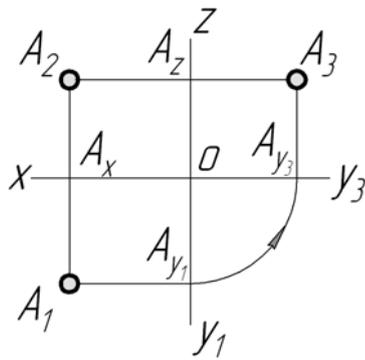


Figure 2.5

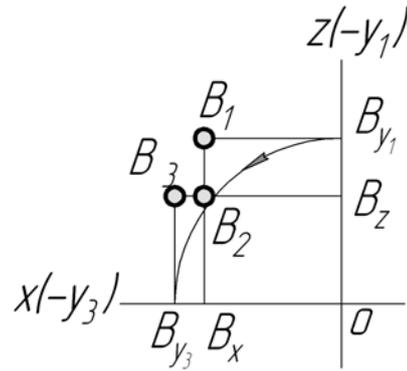


Figure 2.6

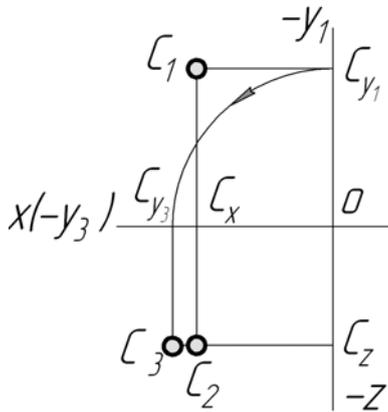


Figure 2.7

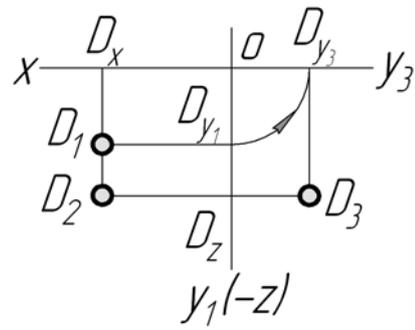


Figure 2.8

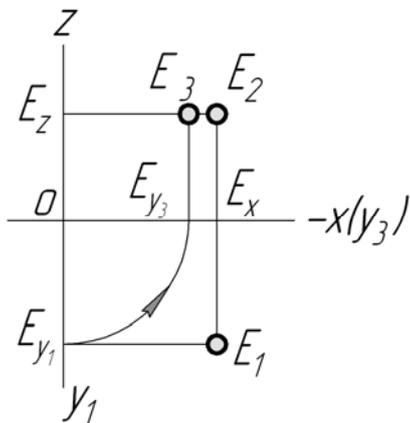


Figure 2.10

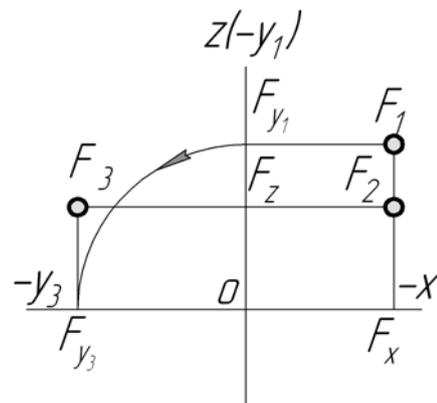


Figure 2.11

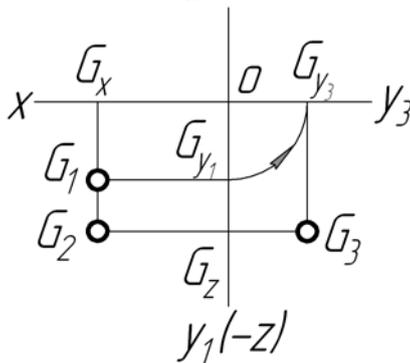


Figure 2.12

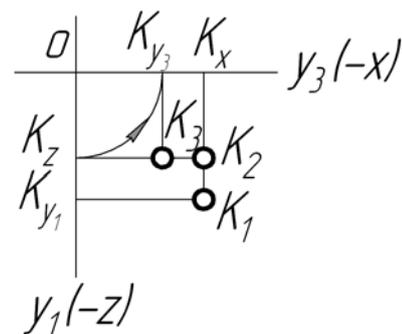


Figure 2.13

2.2 PRINCIPAL PROJECTIONS

The principal projections of object:

projection on the Π_1 is variously called the *horizontal projection*, *top projection*, *top view*, or *plan*;

projection on the Π_2 is called the *front (frontal) projection*, *face-to-face projection* or *front elevation*, *elevation view*;

projection on the Π_3 is called the *profile projection*, *profile* or *side view*.

x axis is often called as the *ground line*.

The line connecting the two projections is called the *projector* or link line, corresponding line.

3 STRAIGHT LINES INTO SYSTEM OF COORDINATE PLANES

A line is the path of a moving point, and is not necessarily straight. Yet in ordinary use, the term **line**, by itself, and without anything to imply the contrary, always means a straight line.

The following statements are evident (reminding of aforesaid):

- the projection of a straight line is a straight line;
- projection of any point of the line lies on the projection of the line.

Since any point in space is definitely determined when its projection on Π_1 and Π_2 are known, it follows then, in general, any two straight lines assumed at random, one in Π_1 and one in Π_2 , are generally the projections of one and the only one straight line in space.

3.1 LOCATION OF THE LINE

The straight line is given using two projections of two points.

The two projections of a line being given, the line is in general completely determined (Fig.3.1).

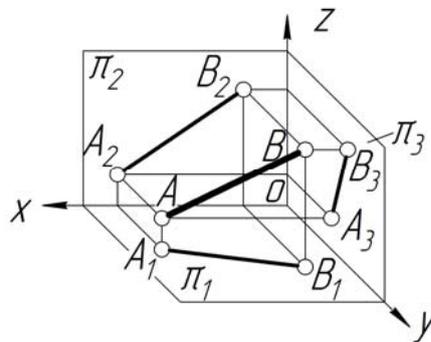


Figure 3.1

Since the projections of a line are made up of the projections of all points of the line, the projection of a line which passes through a given point, passes through the projections of the point.

Among the different positions in which a line may be placed concerning to the coordinate planes, we limit our choice of location to one angle.

The positions in the 1st angle are shown in Fig. 3.2.

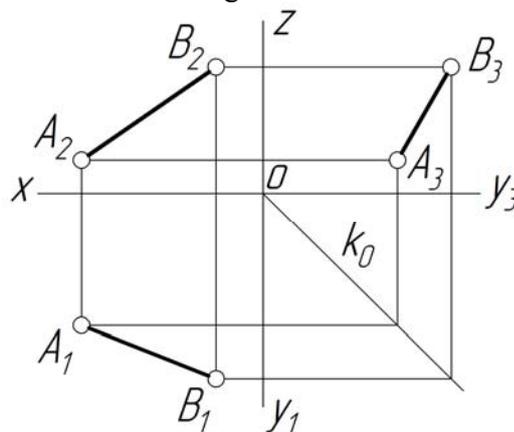


Figure 3.2

3.2 POSITION CLASSIFICATION

Particular position of the line

Line which parallel to one of projection planes is called **level line**: **horizontal**, **frontal** and **profile** lines (Fig. 3.3).

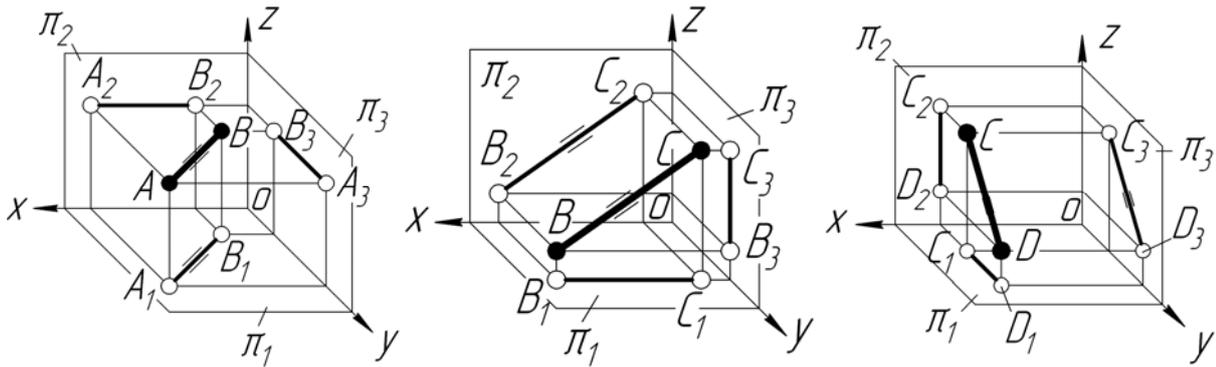


Figure 3.3

Level line: *horizontal, frontal* and *profile* line.

Parallel to Π_1 – frontal projection (on Π_2) is a line, parallel to x ,

horizontal projection (on Π_1) is a line, inclined to axis.

profile projection (on Π_3) is a line, parallel to y_3 .

Parallel to Π_2 – frontal projection (on Π_2) is a line, inclined to axis,

horizontal projection (on Π_1) is a line, parallel to x ,

profile projection (on Π_3) is a line, parallel to z .

Parallel to Π_3 – frontal projection (on Π_2) is a line, perpendicular to x ,

horizontal projection (on Π_1) is a line, perpendicular to x ,

profile projection (on Π_3) is a line, inclined to axis.

Perpendicular to one of projection planes and parallel to two others is called (**projection line**): *horizontal-projecting, frontal-projecting* and *profile-projecting* line (Fig. 3.4).

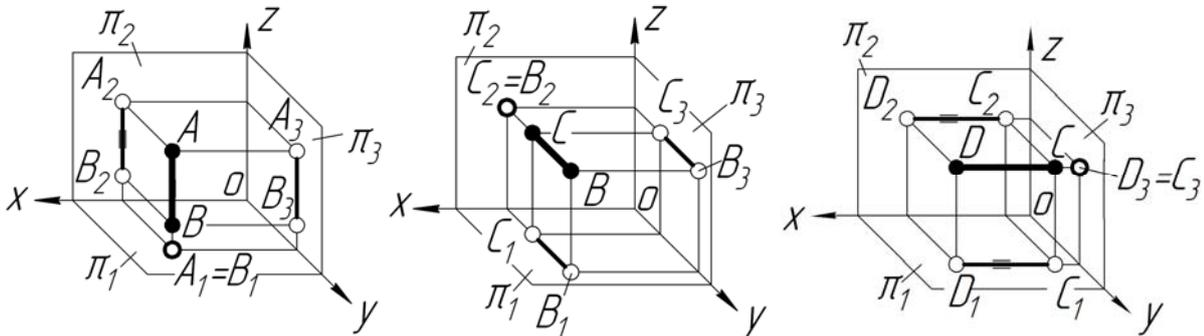


Figure 3.4

Perpendicular to Π_1 – frontal projection (on Π_2) is a line, perpendicular to x ,

horizontal projection (on Π_1) is a point,

profile projection (on Π_3) is a line, perpendicular to y_3 .

Perpendicular to Π_2 – frontal projection (on Π_2) is a point,

horizontal projection (on Π_1) is a line, perpendicular to x ,

profile projection (on Π_3) is a line, perpendicular to z .

Perpendicular to Π_3 – frontal projection (on Π_2) is a line, parallel to x ,

horizontal projection (on Π_1) is a line, parallel to x ,

profile projection (on Π_3) is a point.

Inclined to all projection planes – all projections are *inclined to axis* (or *general provision*) (Fig.3.1, 3.2).

If a line is perpendicular to a plane, its projection on the plane is a point. A line segment parallel to a plane projects in its true length on the plane.

A **vertical (horizontal-projecting)** line projects as a point on the Π_1 –plane (Fig. 3.5). A **horizontal** line segment appears in true length in the top view (on the Π_1 –plane) (Fig. 3.6). The angle between the horizontal projection, $A_1 B_1$, and x is the true angle between line AB and the vertical (frontal) plane. A **frontal** line is defined as a line parallel to the Π_2 –plane (Fig. 3.7). The

front view B_2C_2 is a true-length projection and also shows the true angle between line BC and the horizontal plane (Π_1). A **profile** line is parallel to the plane (Π_2) (Fig. 3.8 -30). The side view B_3C_3 is a **true-length** view. The angle between B_3C_3 and x is the true angle between line BC and the vertical (frontal) plane.

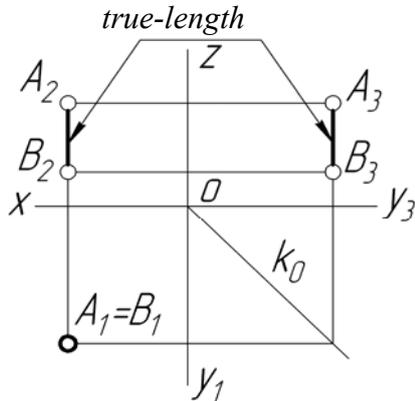


Figure 3.5

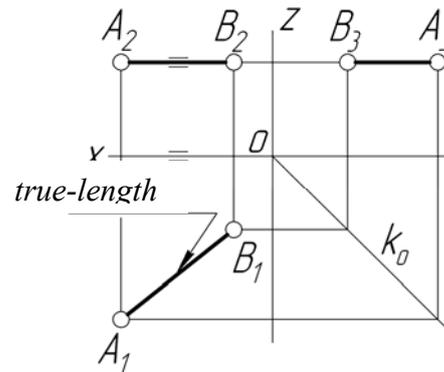


Figure 3.6

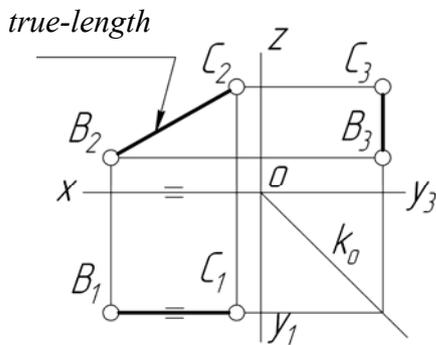


Figure 3.7

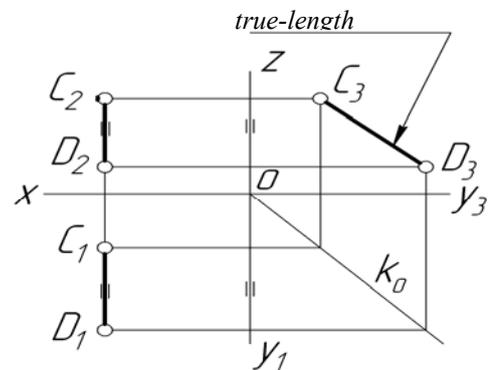


Figure 3.8

It should be noted that when a line segment is parallel to the reference line in one view, the adjacent view is a **true-length** view.

In general there are two parts of the problem solution: the analysis, or the method of the solution, in which the geometrical operations in space which lead to the solution are described; and the construction, or drawing, in which the operations described in the analysis are carried out by the methods of orthographic projection.

3.3 POINT ON A LINE

If a point lies on a line, a pair of the projections of the point will lie on a line which is perpendicular to the reference line.

Since the projections of the point must lie on the corresponding projections of the line and on the same perpendicular to x , assume any point A_2 on the Π_2 projection of CB (Fig. 3.9) and from A_2 drop a perpendicular to x ; the point A_1 in which this perpendicular intersects C_1B_1 is the horizontal projection of point A .

If a space line is divided in a given ratio, its projection is divided into the same ratio. For example, the mid-point of a line segment projects into the mid-point of the projection of the line segment.

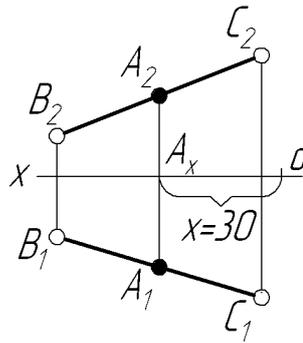


Figure 3.9

3.4 RELATIVE POSITION OF TWO LINES

There are three possible relative position of two straight lines:

- *intersecting lines*,
- *parallel lines*,
- *skew lines*.

The first two types of lines are coplanar lines.

Two intersecting lines

If two lines intersect, their projections will intersect, and the points of intersection of their projections will be the projections of the point of their intersection.

Since the point of intersection is common to both lines, its frontal projection lies on the frontal projection of each line, hence at their intersection; likewise the horizontal projection of the point of intersection is the intersection of the horizontal projections. These two points, being the projections of the same point, must lie on the same perpendicular to x .

Assume that both projections of one line AB (Fig. 3.10) and one projection, C_2D_2 , of the other line CD , intersect A_2B_2 at any point E_2 . From E_2 drop a perpendicular to x until it intersects A_1B_1 at E_1 ; through E_1 draw the horizontal projection of CD in any desired direction.

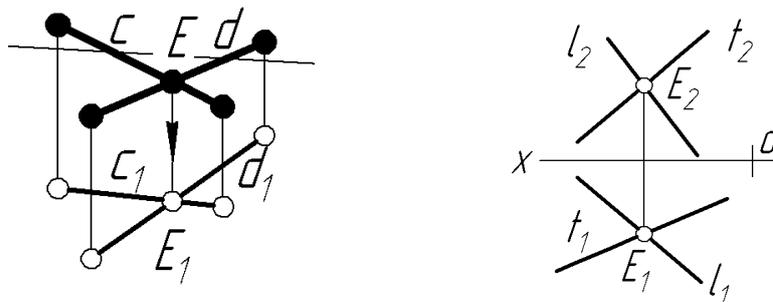


Figure 3.10

Two parallel lines

If two lines are parallel their projections are parallel.

To draw a line through a point and which is parallel to a given line.

If two lines are parallel in space, their projections on a plane are parallel. In general, if the horizontal and frontal projections of two lines are parallel, the lines are parallel in space. Profile lines are an exception and are parallel only when the profile projections of the line are parallel.

Let the given line and point be l and A , respectively (Fig 3.11). Draw the required line k through point A , making k_1 parallel to l_1 and k_2 parallel to l_2 .

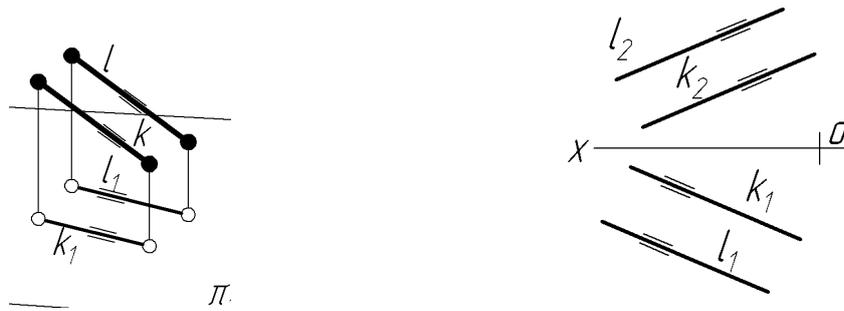


Figure 3.11

Two skew lines

If two lines are skew lines, their projections may be intersected, but the points of intersection of their projections will be no on the link line (Fig. 3.12).

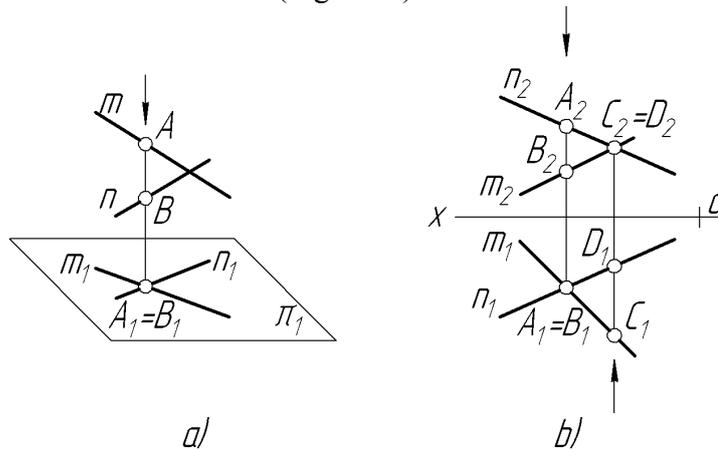


Figure 3.12

3.5 THE TRACES OF A LINE (PIERCING POINTS)

Of all the points in a straight line, the two in which it pierces the planes of projection are considered the most important. These points are called the traces of the line. The horizontal trace is the point in which the line pierces Π_1 , and will be designated as the point M ; the frontal trace, in which the line pierces Π_2 , will be called N . The line changes its position into space (quadrants or octants) in the traces. The traces of a line are named piercing points too.

To find the traces of a straight line.

The solution of this problem depends on direct visualization. General case – line not lying in a profile plane and is not parallel to it.

The horizontal trace (Fig. 3.13). Looking towards Π_2 , Π_1 is seen edgewise as x , the line a appears as a_2 , hence the line a will pierce Π_1 at the point seen as M_2 , where a_2 crosses x . While this projection conveys no idea of the distance of the point in Π_1 from Π_2 , it does single out, to the exclusion of every other point, the Π_1 piercing point of the line. The actual position of the point in Π_1 is found at M_1 by the projector $M_2 M_1$.

The frontal trace (Fig. 3.13). Looking down towards Π_1 , Π_2 is seen edgewise as x , the line a is seen as a_1 , hence the line a is seen to pass Π_2 at the point N_1 . The actual location, N_2 , in Π_2 , is found on a_2 by the projector $N_1 N_2$.

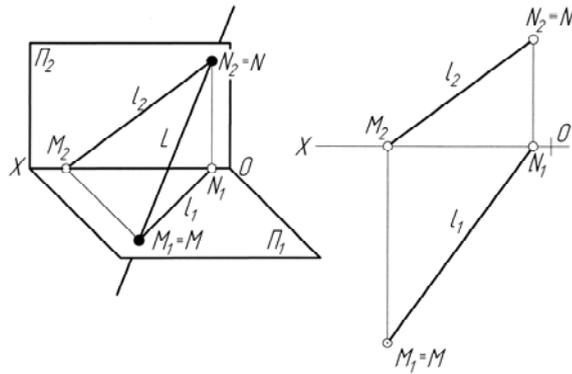


Figure 3.13

Special case is a profile line. The general solution evidently fails; the solution demands profile projection.

In general, a straight line is uniquely determined by its two traces. The only exception is the case in which M and N fall together as a single point on the axis x . The required line can then be any line which passes through this point; in order to determine the line, some other condition must be given (e.g., another point).

Some special lines have no all traces.

3.6 TRUE-LENGTH OF LINE SEGMENT

In general, a projection of line segment is not equal to true-length of line segment. Segment AB is inclined to all planes of projection, therefore segment projections will be less than him.

Let's consider a rectangular triangle $\square ABB_1$. A horizontal projection A_1B_1 it will be equal to a leg of a triangle $\square ABB_1$. To define size of the second leg of a triangle we will look at a face-to-face plane of projections. The projection to a face-to-face plane, B_xB_1 is equal to **full size** of the second leg of a triangle $\square ABB_1$. We will in addition be convinced of it when we will consider private position of straight lines in space. Now running forward, pay your attention, that the leg of a triangle $\square ABB_1$ is perpendicular a horizontal plane of projections and is parallel to a face-to-face plane of projections. In general case **full size** of the second leg of a triangle is $B_xB_{21} - A_xA_2$.

Thus, knowing two legs of a triangle of a rectangular triangle, we can find its hypotenuse. Having the complex drawing of a straight line of the general position where any of projections of a piece of this straight line is not equal to full size of a piece, we can find its full size.

The length of line segment AB may be determined from the right triangle ABB_1 in which one cathetus is equal to the projection of this line segment AB on projection plane (in this case on Π_1) A_1B_1 and another cathetus is equal to difference of the distances from the projection plane to the end points of line segment (Fig. 3.14). Angle (in this case α) in this triangle determines the slope angle of the line segment AB to the projection plane (in this case on Π_1).

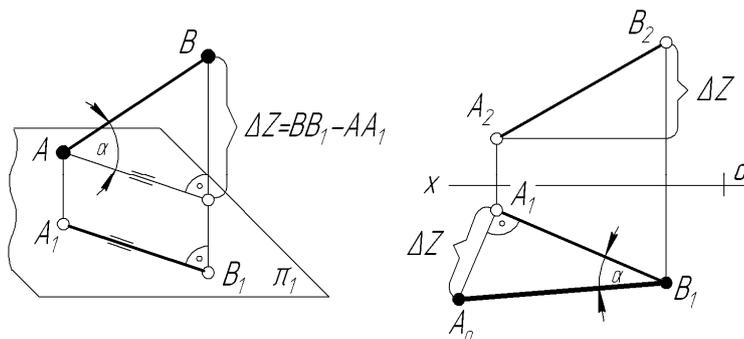


Figure 3.14

The received hypotenuse will be full size of a piece of straight line AB , and the corner α will be a natural corner of an inclination of the given piece to a horizontal plane of projections.

Without a finding natural are long a piece it is impossible to find a corner of an inclination of a straight line to a plane of projections. Therefore if it is required to find corners of an inclination of a straight line to all planes of projections (Π_1, Π_2, Π_3) it is necessary to define natural length of a piece on all planes of projections (Fig. 3.15).

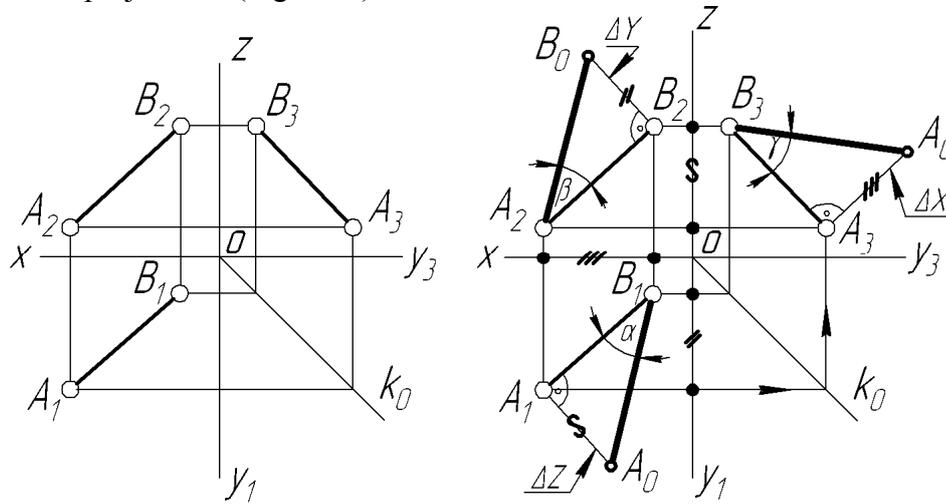


Figure 3.15

3.7 RIGHT ANGLE DISPLAYING

The right angle between two stopped straight lines is projected in the natural size only in that case when one of sides of angle is parallel to a plane of projections. If one party of a right angle is parallel frontal plane of projections the right angle will be projected in the natural size on a face-to-face plane of projections, if one party of a right angle is parallel horizontal plane of projections the right angle will be projected in the natural size on a horizontal plane of projections (Fig.3.16).

It has very much great value at constructions on the complex drawing

- 1) straight lines perpendicular to each other;
- 2) straight line perpendicular to a plane;
- 3) mutually perpendicular planes.

And accordingly, if any of the parties of a right angle does not occupy position level straight line the corner will not be projected full-scale.

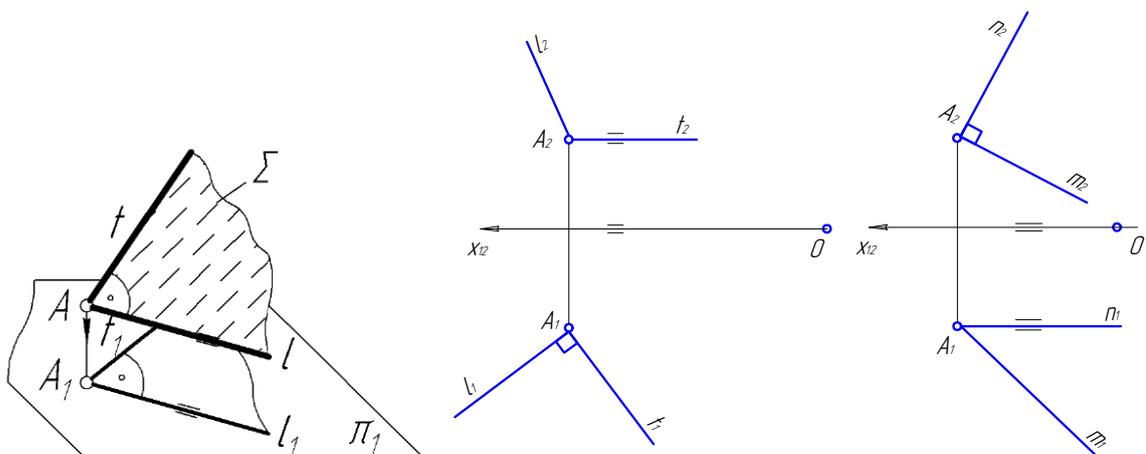


Figure 3.16

4 PLANE INTO SYSTEM OF COORDINATE PLANES

4.1 REPRESENTATION OF THE PLANE

A plane of unlimited extent can be represented on a complex drawing by the projection of three points which are not laying on one straight line; projections of a straight line and the point which is not laying on the given straight line; projections of two parallel straight lines; two crossed straight lines (Fig. 4.1).

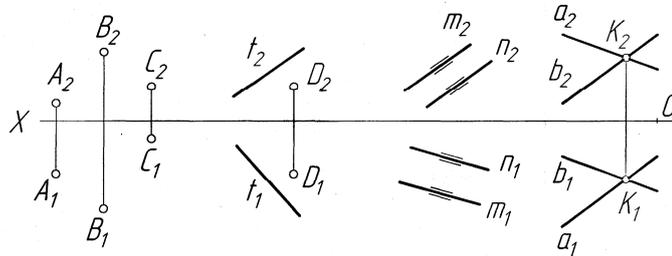


Figure 4.1

In practice, these points are usually joined to form two intersecting lines or triangle, the latter being a convenient form for graphical purposes.

A plane can be uniquely represented by its intersections with the coordinate planes (Fig. 4.2).

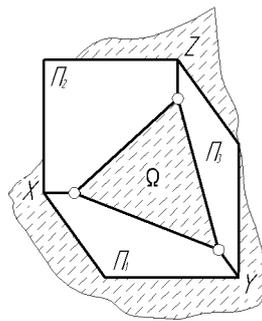


Figure 4.2

4.2 TRACES OF THE PLANE

Lines of crossing of a plane with planes of projections are called the *traces of the plane* (Fig. 4.3).

These lines of intersection of the plane with horizontal Π_1 , frontal Π_2 and profile Π_3 planes of projection are called, respectively, the *horizontal, frontal and profile traces* of the given plane. The horizontal, frontal and profile traces of the plane Ω are designate, respectively, Ω_1 , Ω_2 , and Ω_3 or h_0 , f_0 and p_0 (Fig. 4.3).

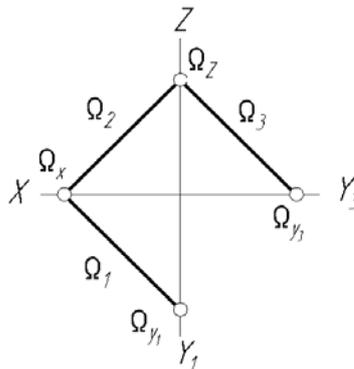


Figure 4.3

The horizontal and frontal traces meet in the point in which the given plane cuts the axis x . The horizontal and profile traces meet in the point in which the given plane cuts the axis y . The frontal and profile traces meet in the point in which the given plane cuts the axis z (Fig. 4.3).

The visualization of a plane from its traces is an entirely different process from reading the projection of a line or a point. The traces of the plane oblique to horizontal Π_1 , frontal Π_2 and profile Π_3 planes of projection are not projections of the plane in the ordinary sense.

Plane is not limited to the portion of the plane in the first angle. Producing the traces through x , we have immediately, the traces of the portions of the plane in all of the other angles. The traces of a plane are not limited by x .

4.3 POSITION CLASSIFICATION

Particular position of the plane

Planes are general positions and particular.

The general provisions plane (oblique) is not parallel and is not perpendicular any of planes of projections.

The plane of *particular position* is parallel or perpendicular at least to one of planes of projections. Planes of particular position share on two groups:

Projecting planes (inclined) - perpendicular to one plane of projection and inclined to two another.

Level planes are perpendicular to two planes of projection and parallel to another of them.

Projecting planes can be in three positions:

Plane is perpendicular a horizontal plane of projection and an inclination to face-to-face and profile planes (Fig. 4.4). It is called *horizontal-projecting* plane;

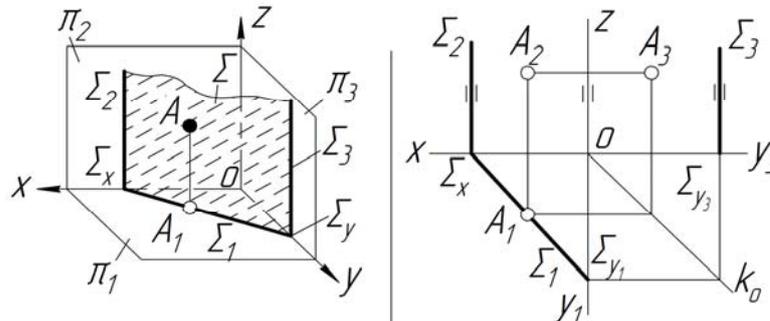


Figure 4.4

The frontal and profile traces of this plane are parallel to the axis z .

Plane is perpendicular a face-to-face plane of projection and an inclination to horizontal and profile planes (Fig. 4.5). It is called *frontal-projecting* plane;

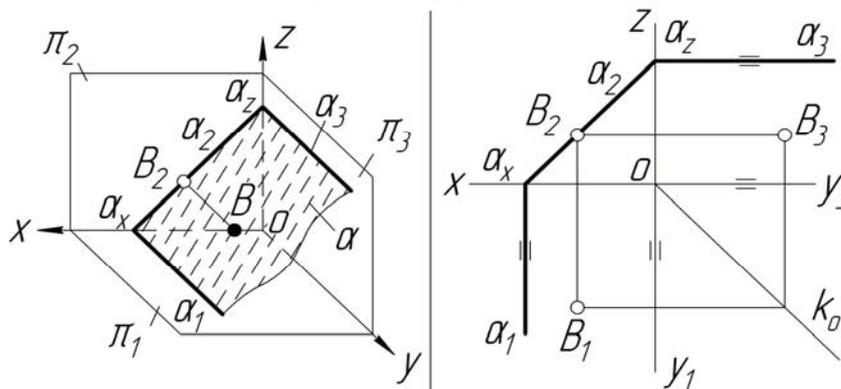


Figure 4.6

The horizontal and profile traces of this plane are parallel to the axis y .

Plane is perpendicular a profile plane of projection and an inclination to horizontal and face-to-face planes (Fig. 4.7). It is called **profile-projecting** plane.

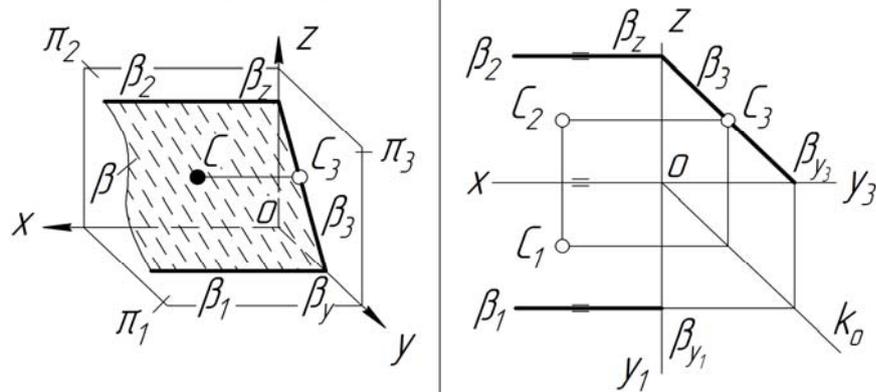


Figure 4.7

The frontal and horizontal traces of this plane are parallel to the axis x .

Level planes also can be in three positions:

Plane is parallel to a horizontal plane of projections and it is perpendicular face-to-face and profile (Fig. 4.8). It is called **horizontal plane**;

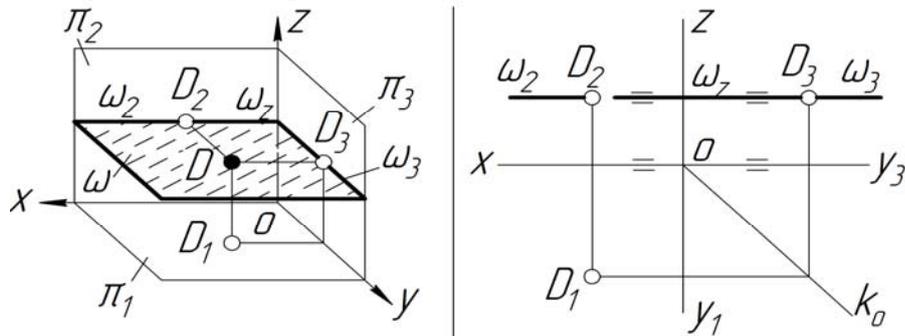


Figure 4.8

If a plane is parallel to horizontal plane of projections Π_1 , there is no its horizontal trace h_0 , while frontal trace f_0 is parallel to the axis x , and conversely.

Plane is parallel to a face-to-face plane and it is perpendicular horizontal and profile (Fig. 4.9). It is called **frontal** plane.

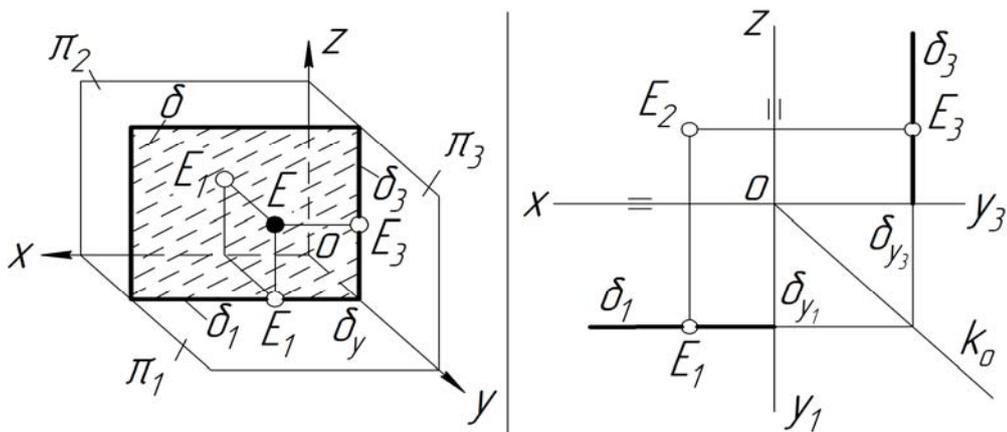


Figure 4.9

If a plane is parallel to frontal plane of projections Π_2 , there is no its frontal trace f_0 , while horizontal trace h_0 is parallel to the axis x , and conversely.

Plane is parallel to a profile plane of projections and it is perpendicular to horizontal and face-to-face planes of projections (Fig. 4.10). It is called **profile plane**.

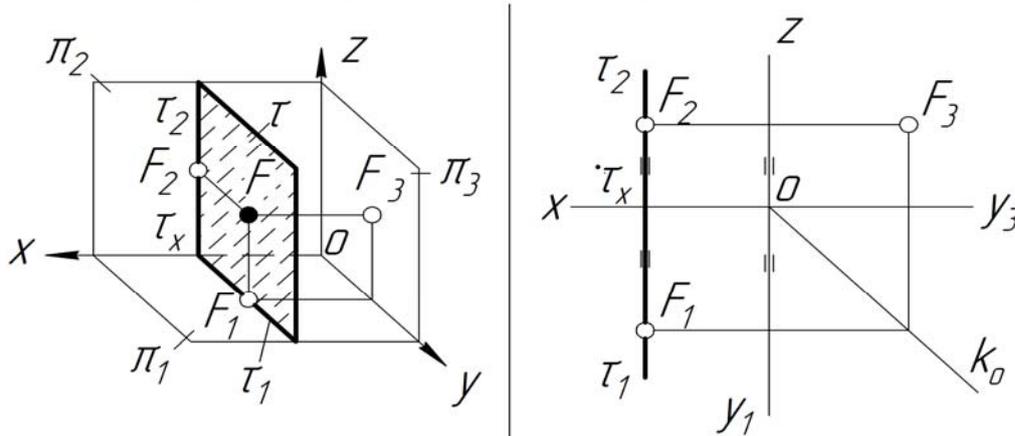


Figure 4.10

If a plane is parallel to profile plane of projections Π_3 , therefore it is perpendicular frontal Π_2 and horizontal Π_1 plane of projections, there is no its profile trace p_0 , while horizontal trace h_0 and frontal trace f_0 are perpendicular to the axis x , and conversely.

The particular position of a plane in which it becomes perpendicular to one of the coordinate planes is an important one to visualize. If plane is perpendicular to horizontal plane of projections Π_1 , then the horizontal projection of every point in this plane must fall in the horizontal trace of this plane h_0 , which thus becomes an actual projection of the plane (Fig. 4.4). The trace h_0 is, in fact, an edge view of the plane. It should be noted that a plane parallel to one of the coordinate planes is necessarily perpendicular to the others.

Planes of level and projecting planes are characteristic that projections of all points and lines laying in these planes, will lay on a projection of this plane which is represented by a straight line.

4.4 POINT AND STRAIGHT LINE IN A PLANE

The point lies on a plane if lies on a straight line lying plane (Fig.4.11).

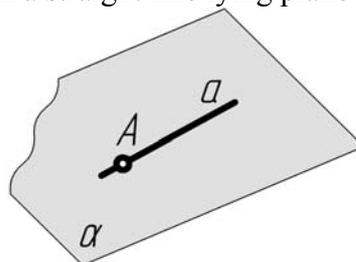


Figure 4.11

The line lies in a plane if its two points lie in this plane.

If a line m lies in a given plane Ω , it must pierce the frontal plane of projections Π_2 in a point common to both planes (Ω and Π_2), therefore in a point on their line of intersection f_0 , the frontal trace N . For similar reasons it must pierce horizontal plane of projections Π_1 in a point on the horizontal trace h_0 . Hence to assume a line in a given plane assume any point on the Π_2 trace as the Π_2 piercing point M (Fig. 4.12).

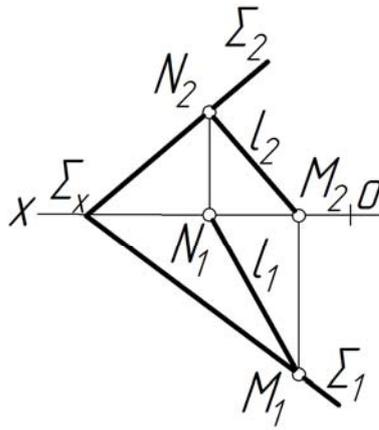


Figure 4.12

Thus if a line lies in a given plane, its traces lie on the respective traces of the plane, and conversely.

If one projection of a line which lies in a plane is given or assumed, the other projection can be found. Let the horizontal projection of line l , lying in the plane determined by lines m and n , be given (Fig. 4.13). Line l_1 cuts m_1 and n_1 in points A_1 and B_1 respectively. The frontal projection of l is determined by points A_2 and B_2 .

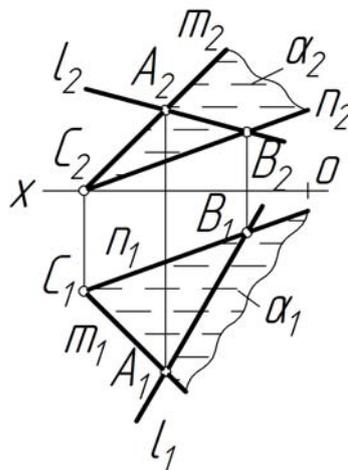


Figure 4.13

Let the plane \mathcal{A} is determined by parallel straight lines a and b . A frontal projection of point A_2 is given, it is necessary to construct A_1 (Fig. 4.14).

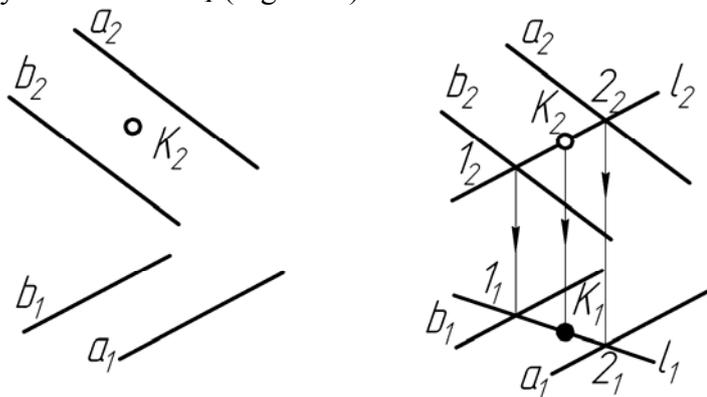


Figure 4.14

4.5 THE PRINCIPAL LINES OF A PLANE

The lines of a plane which are parallel to the planes of projections are the principal lines of the plane.

Any line which lies in a given plane and is parallel to Π_1 , is called a **horizontal** of that plane. Being parallel to Π_1 , it has not one horizontal trace, and its frontal projection is parallel to x . Obviously the horizontal projection of a horizontal is parallel to the horizontal trace of the plane. Horizontal trace of the plane is named **zero-level horizontal** of the plane too.

A horizontal h of a plane may be assumed by assuming its Π_2 piercing point N , and drawing its frontal projection h_2 through N_2 parallel to x and the Π_1 projection h_1 thru N_1 parallel to the horizontal trace h_0 (Fig. 4.15).

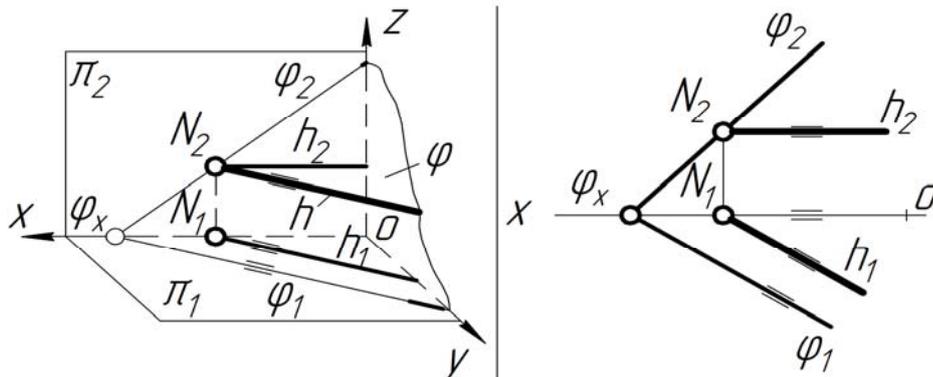


Figure 4.15

Any line which lies in a given plane and is parallel to Π_2 , is called a **frontal** of that plane. Being parallel to Π_2 , it has not one frontal trace, and its horizontal projection is parallel to x . Obviously the frontal projection of a frontal is parallel to the frontal trace of the plane. Frontal trace of the plane is named zero-level frontal of the plane too.

A frontal f of a plane may be assumed by assuming its Π_1 piercing point M , and drawing its horizontal projection f_1 through M_1 parallel to x and the Π_2 projection f_2 thru M_2 parallel to the frontal trace f_0 (Fig. 4.16).

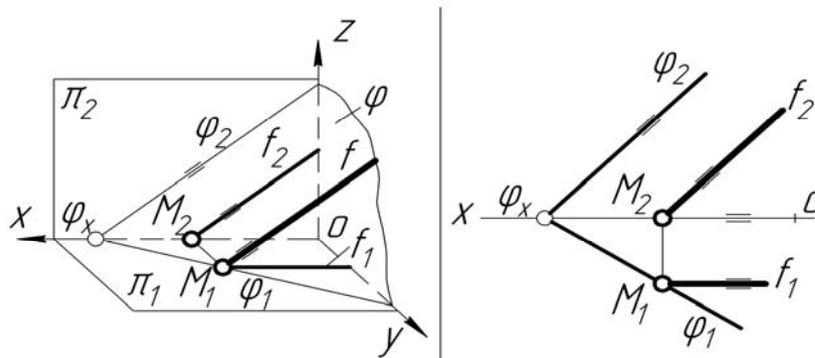


Figure 4.16

A horizontal h and frontal f of a plane may be assumed by founding projections of its two points which lie in given plane if plane is represented by another way (Fig. 4.17).

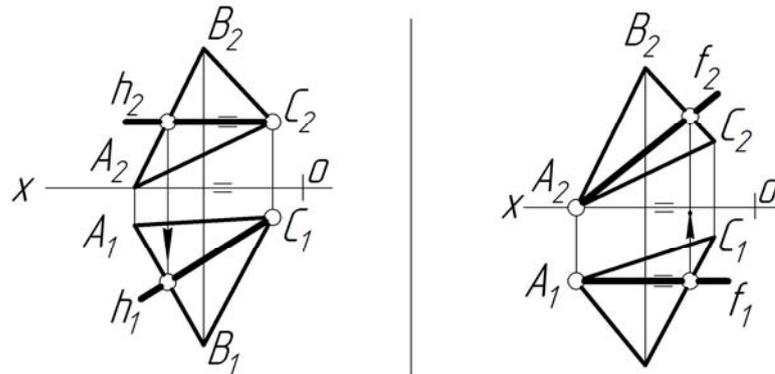


Figure 4.17

In general, through a given point of a plane one horizontal and one frontal line can be drawn.

Steepest line (lines of maximum inclination)

The lines of a plane which have the greatest inclination to the plane of projection name lines of maximum inclination or maximum slope line.

Lines of maximum inclination to the horizontal plane of projections Π_1 are perpendicular to the horizontal lines of the plane (Fig. 4.18).

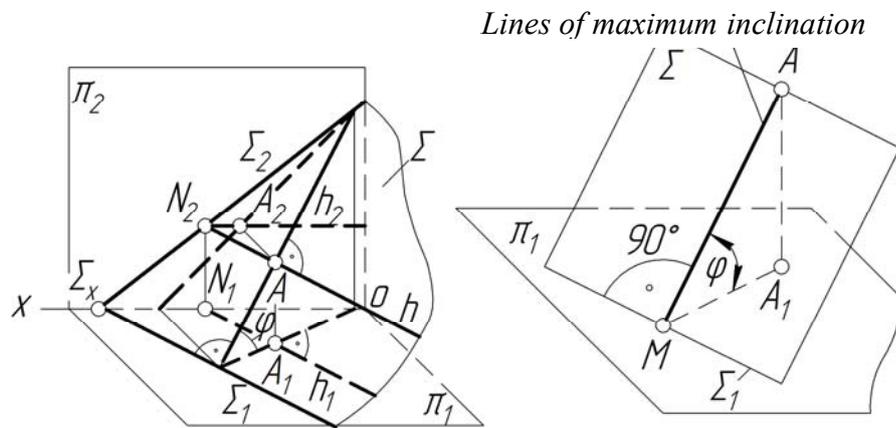


Figure 4.18

4.6 PARALLEL RELATION BETWEEN LINES AND PLANES

A line is parallel to a plane if it is parallel to some line in the plane. Converse, a plane is parallel to a line if the plane contains a line parallel to the given line (Fig. 4.19).

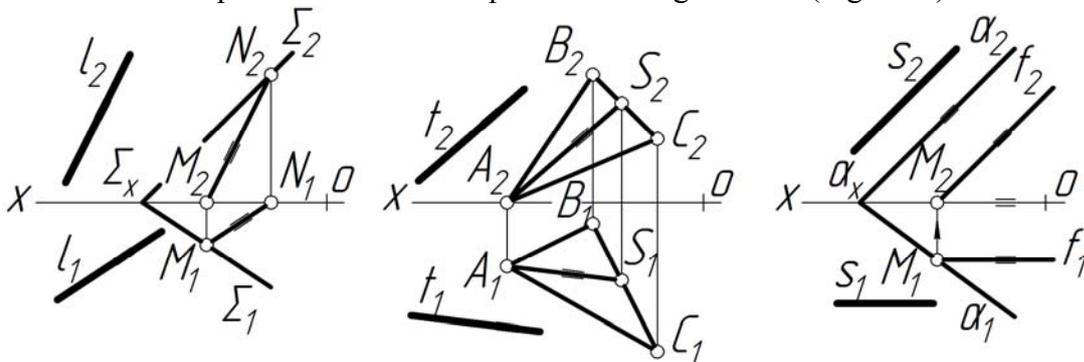


Figure 4.19

4.7 PARALLEL PLANES

Two planes are parallel if two intersecting lines in one plane are parallel to two intersecting lines of other plane (Fig. 4.20).

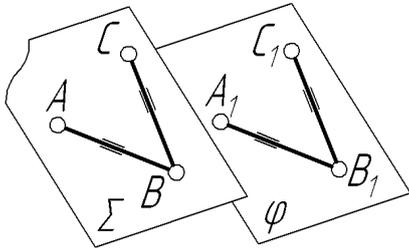
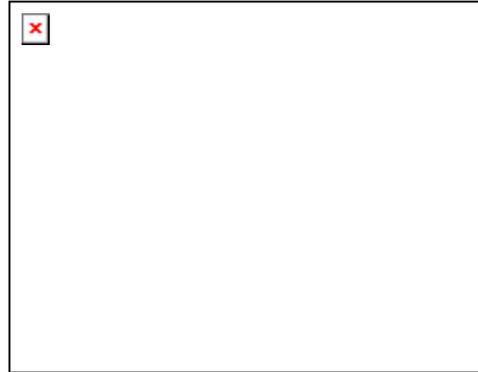


Figure 4.20



If two planes are parallel their respective traces are parallel. If two parallel planes cut by a third plane, the lines of intersection are parallel (Fig. 4.21 -58).

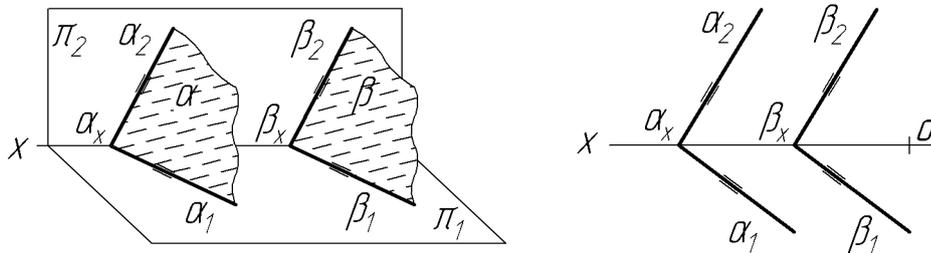


Figure 4.21

If two respective traces are parallel the respective principal lines are parallel too.

4.8 INTERSECTING PLANES

The line of intersection of two planes is a line common to both planes; therefore its horizontal piercing point lies in the horizontal trace of each plane, hence at their point of intersection. The frontal piercing point of the line of intersection lies at the intersection of frontal traces. The line joining these two piercing points is the required line of intersection (Fig. 4.22).

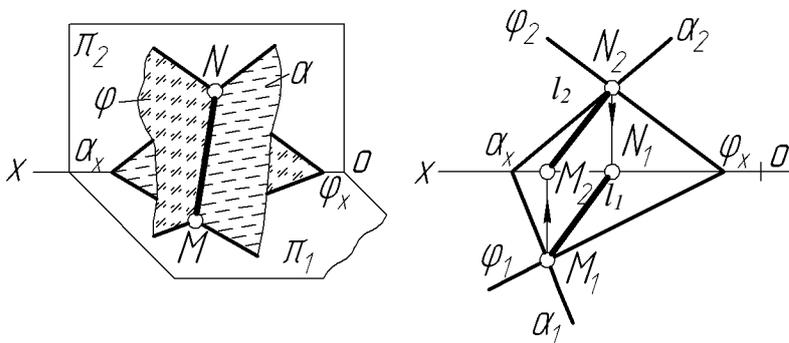


Figure 4.22

Special case of line of intersection of planes

Either pair of traces does not intersect on limits of drawing.

Draw an auxiliary plane parallel to the plane of projections. This plane will cut from the given planes, lines which will intersect in a point of the required line of intersection. A second such

auxiliary plane will in the same manner give a second point on the required line. The line passing through these two points is required line of intersection.

In Figure 4.23 auxiliary plane is parallel to the horizontal plane of projections.

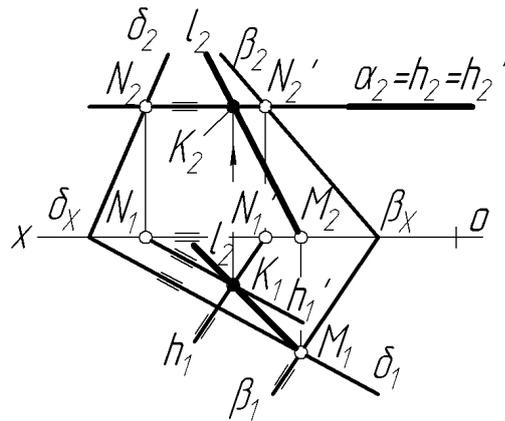


Figure 4.23

All traces intersect on axis x

In this case both the horizontal and frontal piercing points of the line of intersection are at the intersection of the traces with x hence both projections of the line of intersection pass through this point (Fig 4.24).

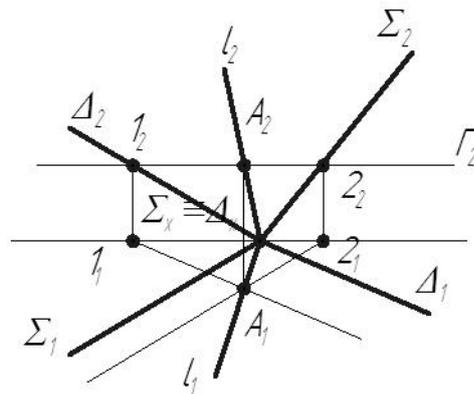


Figure 4.24

Two traces of planes are parallel

In this case the line of intersection is parallel to the pair of parallel traces, hence respective projection of the line of intersection is parallel to their (Fig 4.25).

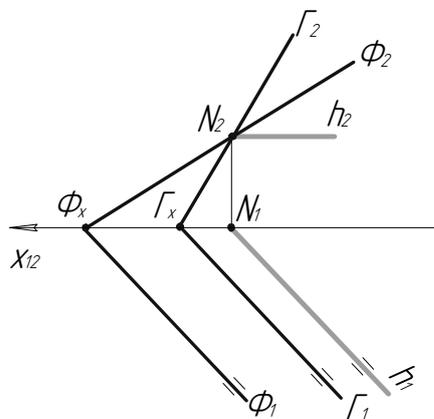


Figure 4.25

Both planes are profile-projecting

Profile-projecting plane is parallel to the axis x (Fig.4.26). In this case horizontal and frontal traces are parallel to the axis x too. One point of the line of intersection may be obtained by finding its profile trace on profile plane of projection. Find the profile traces of the given planes. Their intersection gives P_3 , the profile trace of the required line of intersection planes. From P_3 obtain P_1 and P_2 , one point on the line of intersection. A second point is not necessary, since this line must be parallel to both Π_1 and Π_2 , that is, parallel to the axis x .

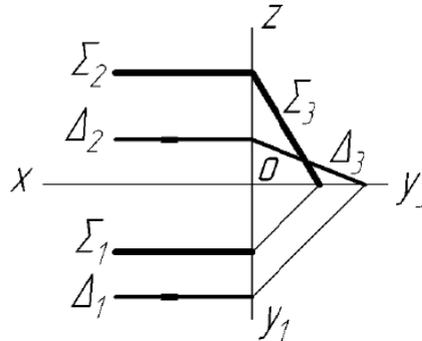


Figure 4.26

4.9 THE INTERSECTION OF A LINE AND A PLANE

Let a line l intersect a plane Ω (Fig 4.27). The point of intersection will be determined if we find where l intersects a line in plane Ω . The line cannot, however, be any line chosen random in Ω , for such a line will probably not intersect l . Let a plane, Δ , be passed through the line l . Then Δ will intersect Ω in a line, m . This line m is a line in the plane Ω , which is intersected by l at the point K . Hence K is the required point in which the line l intersect plane the Ω .

Horizontal-projecting or frontal-projecting auxiliary plane Δ is the most convenient for task solution.

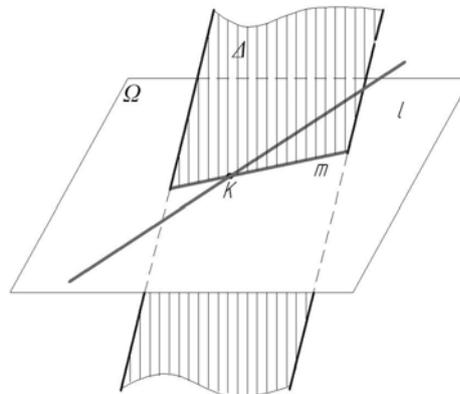


Figure 4.27

The intersection of a line and an oblique plane can be determined by using a vertical (horizontal-projecting) or frontal-projecting cutting plane which contains the given line. The line of intersection of the cutting plane with the oblique plane and the given line must intersect or be parallel because they both lie in the vertical cutting plane (Fig. 4.28). Since the cutting plane appears as an edge in the plan view (horizontal projection), the relationship between the line of intersection and the given line is not apparent in the plane view. The related view, however, reveals this relationship. Should the two lines intersect in the related view, it is evident that the point of intersection is common to both the given plane and the given line and therefore determines the pierce point of the given line and the given plane.

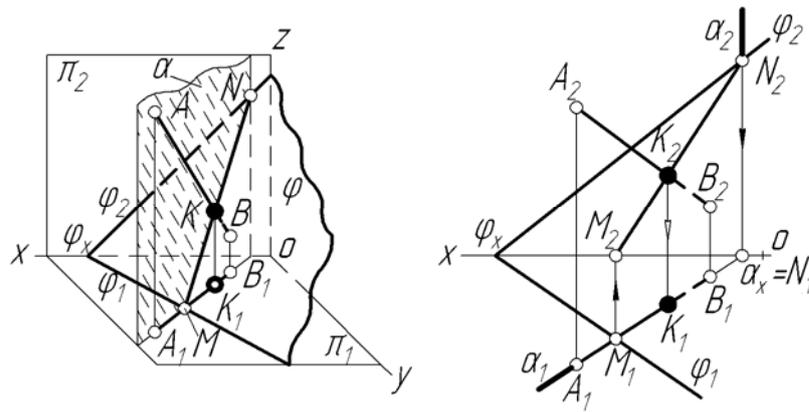


Figure 4.28

In Fig. 4.29 the oblique plane ABC and the line l are given in both the horizontal and frontal projections. A frontal-projecting plane Δ , coincidental with and containing the given line l , appears as an edge in the frontal projection. The intersection of the given plane ABC and the frontal-projecting cutting plane Δ containing l is the line $1\ 2$. The lines l and $1\ 2$ both lie in the frontal-projecting cutting plane Δ and intersect each other at point K in the horizontal projection. Since point K is on line l , it is also on plane ABC because line $1\ 2$ is on plane ABC . Therefore point K is the required point, being common to both the line l and the given plane ABC . It can now be projected to the related projection. Use careful visualization to determine what portion of the line should be visible in each projection.

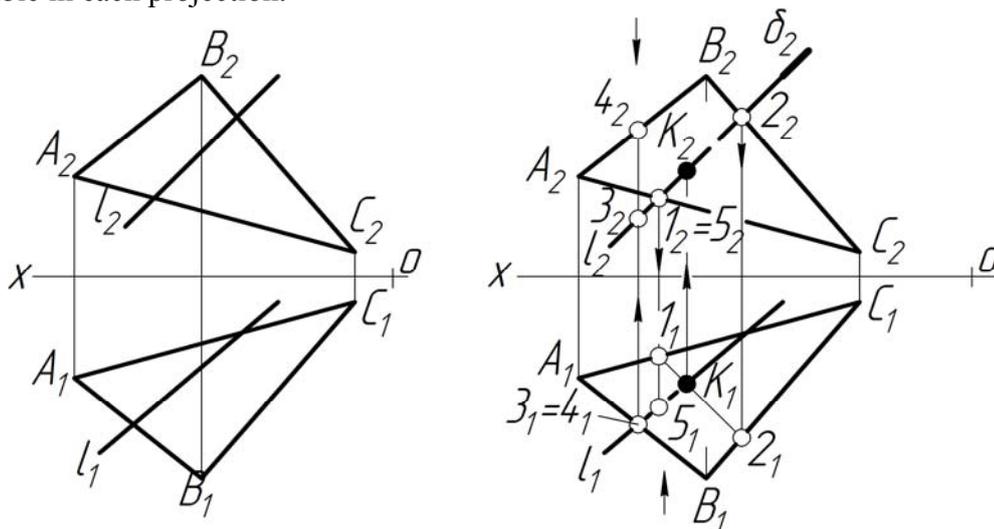


Figure 4.29

If line $1\ 2$ had appeared parallel to l in the horizontal projection, it would have indicated that the line l was parallel to plane ABC and therefore it would have no point of intersection with the given plane.

4.10 DETERMINING THE VISIBILITY OF LINES ON THE PROJECTIONS

To determine which line of an apparent intersection of two lines is more distant to the horizontal projection, project the exact crossover point of the lines to the adjoining frontal projection.

In the adjoining frontal projection, determine which of the lines is more distant to the axis x . This line is above of the other line in the horizontal projection and is visible in this projection.

To determine which line of an apparent intersection of two lines is more distant to the frontal projection, project the exact crossover point of the lines to the adjoining horizontal projection.

In the adjoining horizontal projection, determine which of the lines is more distant to the axis x . This line is in front of the other line in the frontal projection and is visible in this projection.

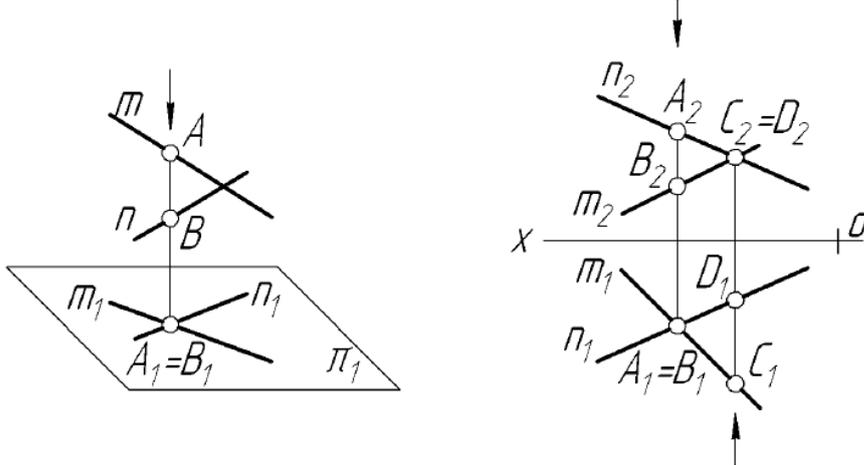


Figure 4.30

In Fig. 4.31 the given plane is perpendicular to Π_1 (horizontal-projecting). No construction is necessary, since Σ_1 is an edge view of the plane, and in the frontal projection the point in which the line l pierces the plane appears directly. The frontal projection of this point is obtained by projecting from the horizontal projection.

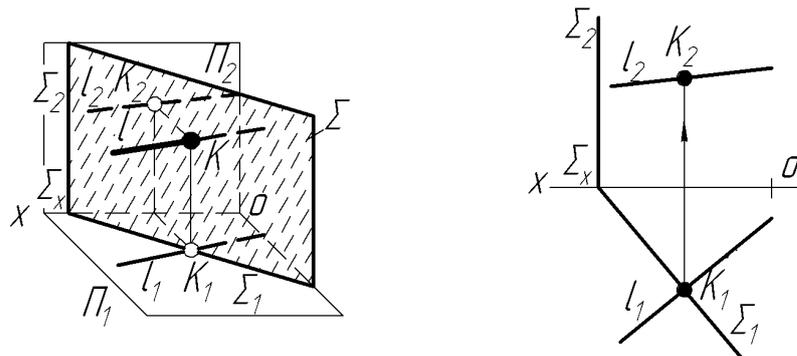


Figure 4.31

4.11 THE INTERSECTION OF TWO LIMITED PLANES

The line of intersection of two planes is a straight line, for the construction of which it is sufficient to determine two points that are common to the two planes, or one point and the direction of the line of intersection of the planes.

Let it be required to find the intersection of the triangle ABC with the plane, indefinite in length, but limited in width by the parallel lines k and s (Fig. 4.32). The intersection can be found, without finding the traces of either plane, by applying the preceding method, as follows. Using the auxiliary plane α which contain s and is perpendicular to Π_1 (horizontal-projecting plane), we find that the line s intersects the plane of the lines AB and AC in the point N (line 1-2 (1_1-2_1 , 1_2-2_2) is the line of intersection of auxiliary plane α with the plane of the lines AB and AC). Using the auxiliary plane γ which contain k and is perpendicular to Π_1 , we find that the line k intersects the plane of the lines AB and AC in the point M (line 3-4 (3_1-4_1 , 3_2-4_2) is the line of intersection of auxiliary plane γ with the plane of the lines AB and AC).

If both the planes of Fig. 4.32 are considered to be opaque, each of them must hide a portion of the other. The visibility of either projection must be determined by means of information obtained from the other projection.

Thus, to determine the visibility of the horizontal-projection, take any point which intersect the projections of two lines in is not in the same plane. For example, consider the point where k

intersects AB (k_1 intersects A_1B_1). This is actually the projection of two points, 3 in the line AB (3_1 in the line A_1B_1), and 5 in k (5_1 in k_1). Project these points to the Π_2 , obtaining 3_2 and 5_2 . From these projections we see that the point 3 (3_2) is higher than the point 5 (5_2); that is, AB passes above k . Therefore, in the Π_1 -projection, A_1B_1 which contains 3_1 , is the visible line at the point under consideration.

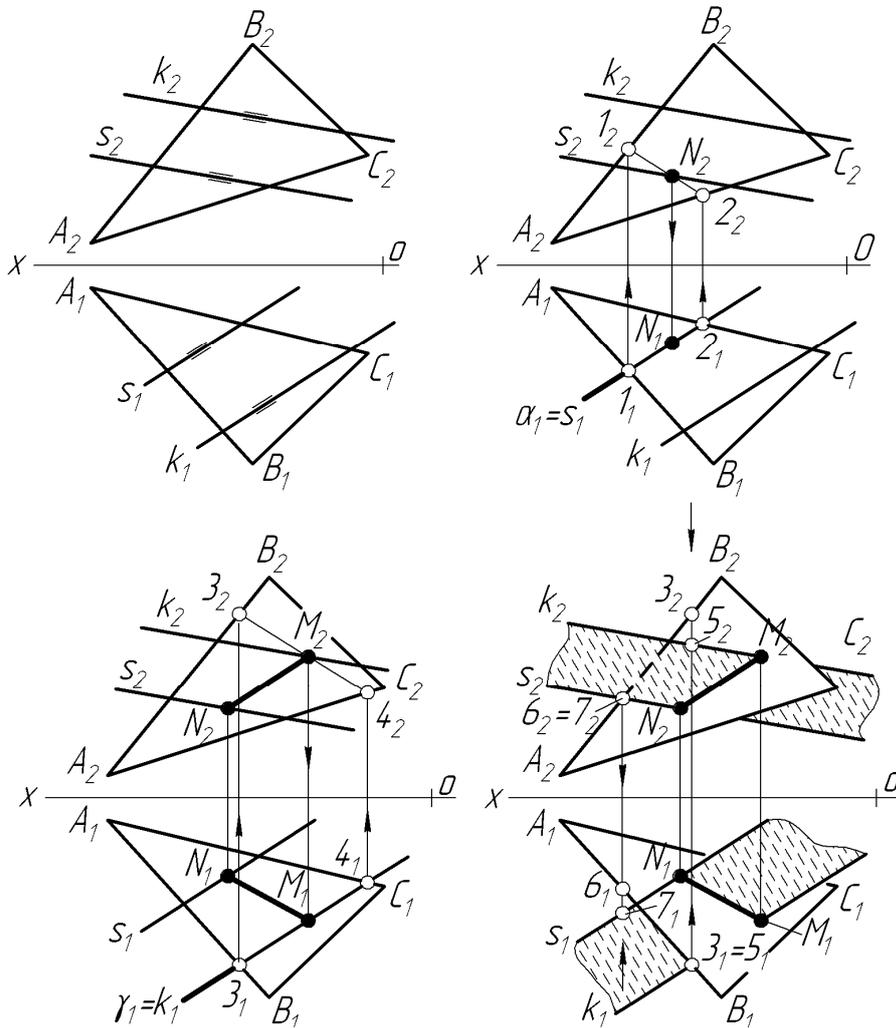


Figure 4.32

We may reason also as follows: the line k has been found to be below the triangle at the point 5 . Now k intersects the triangle at the point M . Therefore beyond M the line k passes out of sight above the triangle, so that at the point where k_1 intersects A_1C_1 , the latter must be the invisible line; and so on around.

The visibility of the frontal projection is similarly determined. Begin at any point where the two projections of lines not in the same plane cross each other, as for instance where s_2 intersects A_2B_2 . Project the horizontal projection of these points to find which line is in front of the other. In this case, point 7 in s in front of point 6 in AB , therefore s_2 is visible. And so on until the complete visibility is found.

4.12 A LINE PERPENDICULAR TO THE PLANE

Before beginning to consider perpendicular to the plane it is well also to remind that a line is perpendicular to the plane if it is perpendicular to any two intersecting straight lines which are laid in this plane.

The right angle between two stopped straight lines is projected in the natural size only in that case when one of sides of angle is parallel to a plane of projections. If one party of a right angle is

parallel frontal plane of projections the right angle will be projected in the natural size on a face-to-face plane of projections, if one party of a right angle is parallel horizontal plane of projections the right angle will be projected in the natural size on a horizontal plane of projections.

Therefore we will take horizontal and frontal principal lines of the plane as any two stopped straight lines.

If a line perpendicular to a plane, then the projections of the line are perpendicular to like traces of the plane (and also to the respective projections of horizontal and frontal principal lines of the plane).

In Fig. 4.33 we have a line AB perpendicular to the plane Σ . Let this line intersect Σ in point B . Draw the horizontal principal line BN (h) through point B in plane Σ . Then, by hypothesis, AB is perpendicular to BN , and, invoking the theorem on the projection of a right angle, we may state that A_1B_1 is perpendicular to B_1N_1 (h_1). Now since Σ_1 (h_0) is parallel to B_1N_1 (h_1), A_1B_1 is perpendicular to Σ_1 (h_0). Similarly, passing a frontal principal line f through point B in plane Σ , we may prove that A_2B_2 is perpendicular to the frontal projection of the frontal principal lines f_2 and to the frontal trace Σ_2 (f_0) of the plane.

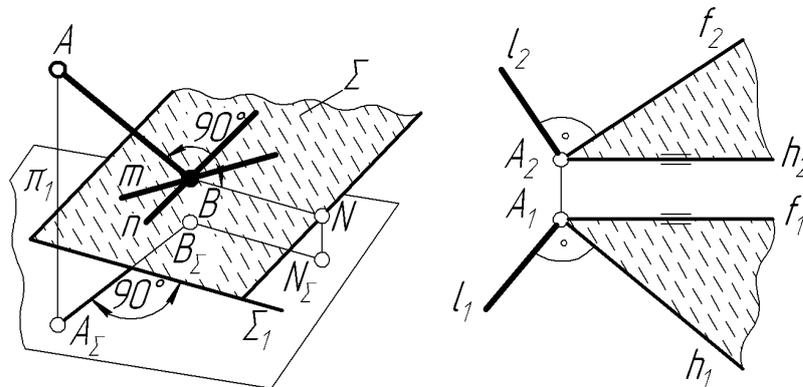


Figure 4.33

The converse statement is also correct. If the projections of the line are perpendicular to like traces of the plane, then this line is perpendicular to a plane.

Fig. 4.34 shows the construction of projections of a perpendicular dropped from a given point A to plane Σ . The same problem is solved in Fig. 4.35 when a plane is given by the triangle ABC and point D . Here the direction of the projections of the perpendicular was determined by the principal lines h and f of the plane of the triangle. Thus, the horizontal projection of the perpendicular is drawn at a right angle to the like projection of the horizontal principal line h_1 , the second projection of the perpendicular is situated at a right angle to the frontal projection f_1 of the frontal principal line f .

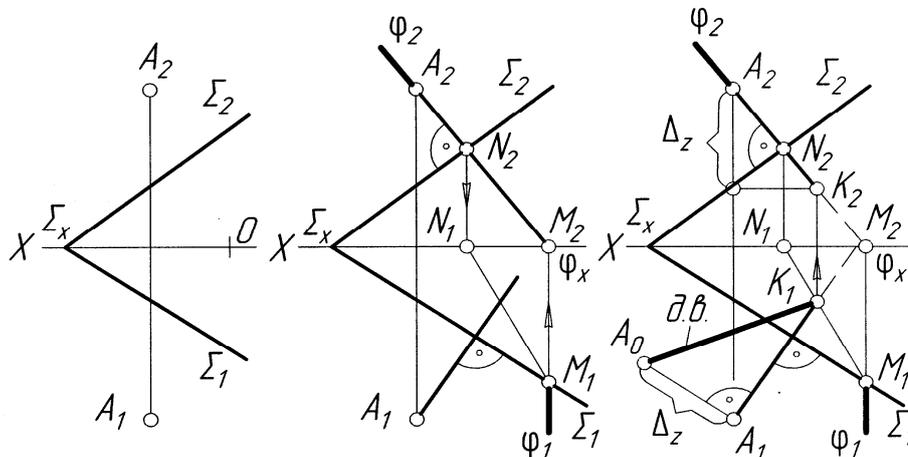


Figure 4.34

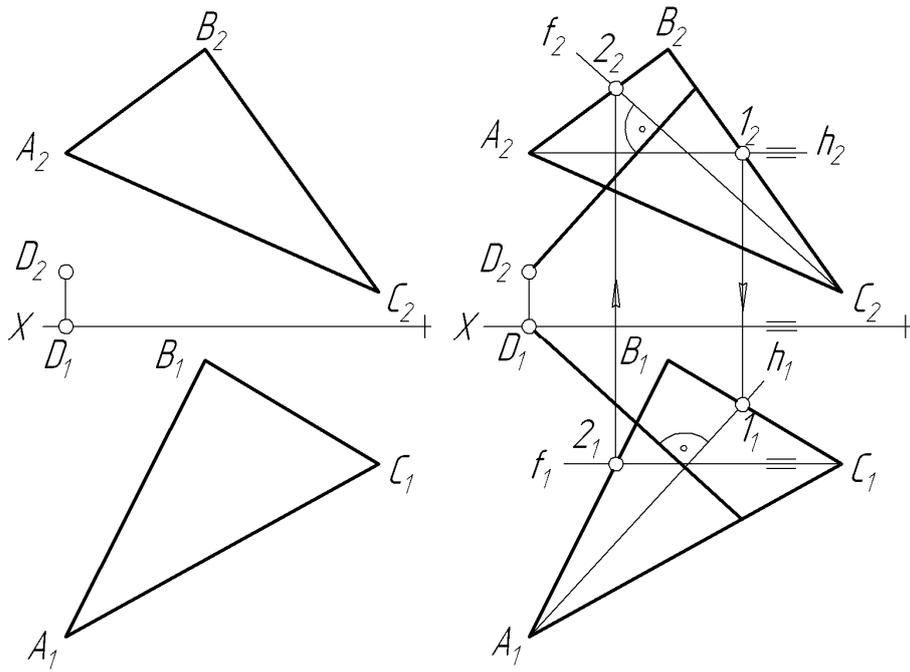


Figure 4.35

4.13 THE SHORTEST DISTANCE FROM A POINT TO A PLANE

The shortest distance from a given point to a given plane may be obtained by dropping a perpendicular from the point to the plane, and then measuring the length of this perpendicular.

From the given point drop a perpendicular to the given plane.

Find the foot the perpendicular, that is, the point in which the line pierces the given plane.

Obtain the true length of the perpendicular.

Let A be the given point, and Σ the given plane (Fig. 4.34). From A draw the indefinite line n , perpendicular to Σ . Find the point, K , in which n intersects Σ (previous problems); in the figure, this is done by using the auxiliary plane ϕ , perpendicular to Π_2 (frontal-projecting). Then A_1K_1 and A_2K_2 are the projections of the required shortest distance, the true length of which may be found by previous problems.

The same problem is solved in Fig. 4.36 when a plane is given by the triangle ABC and point $-D$.

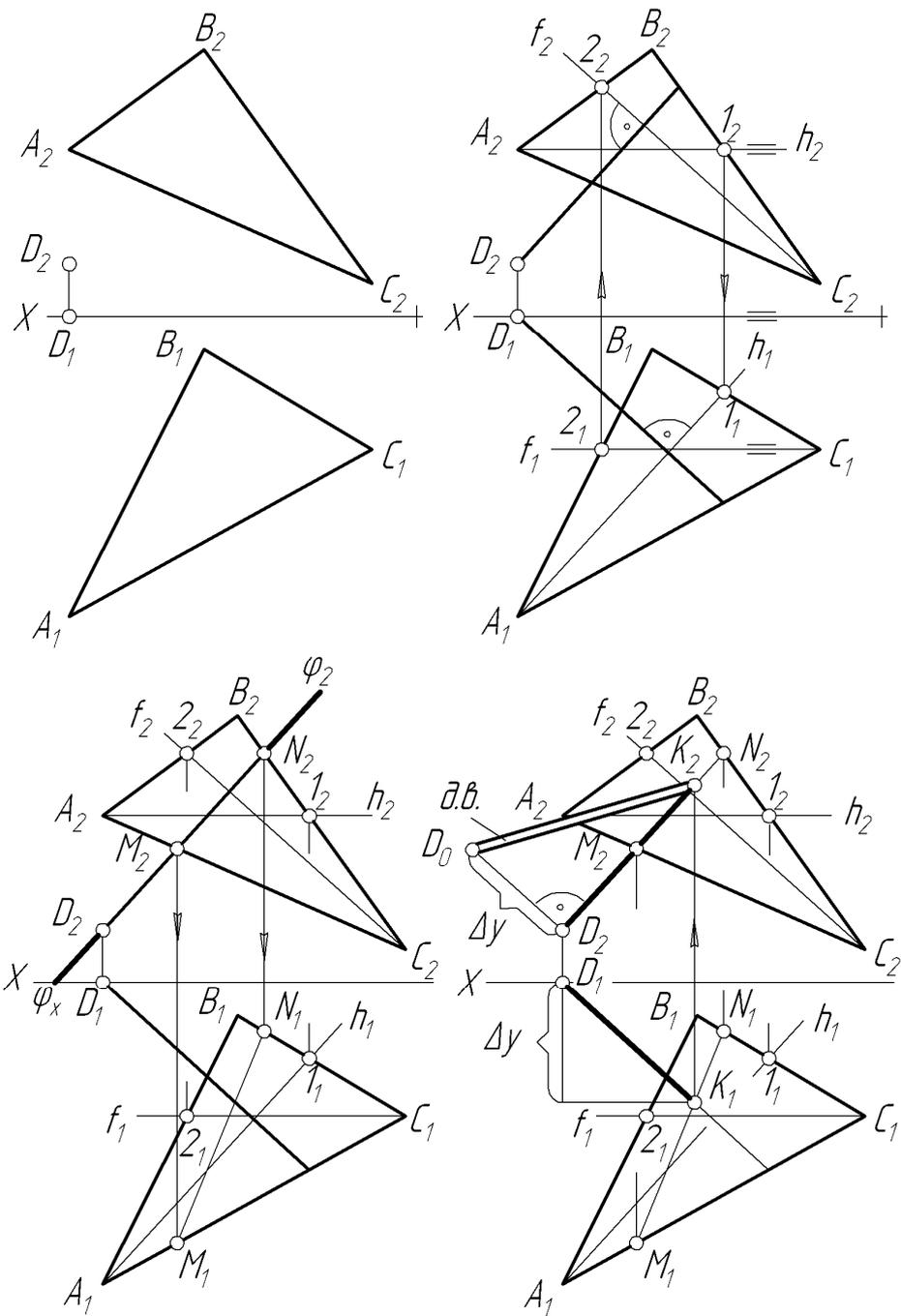


Figure 4.36

Special case. The given plane Σ is parallel to the x (profile-projecting) (Fig. 4.37). The required perpendicular from given point K is evidently a profile line, and may be drawn directly in the profile projection as soon as the profile views of the given point and plane are obtained. In this case it is not necessary to have the horizontal and frontal projections of the perpendicular in order to know its actual length; these projections are, however, usually considered a part of the problem, and are obtained by projecting from the profile view.

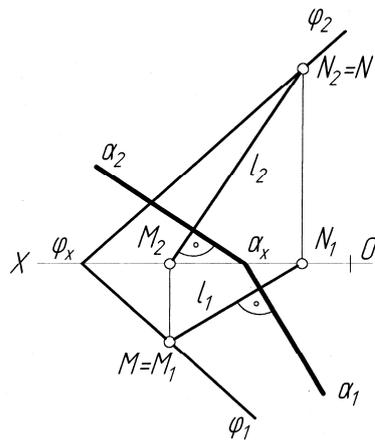


Figure 4.39

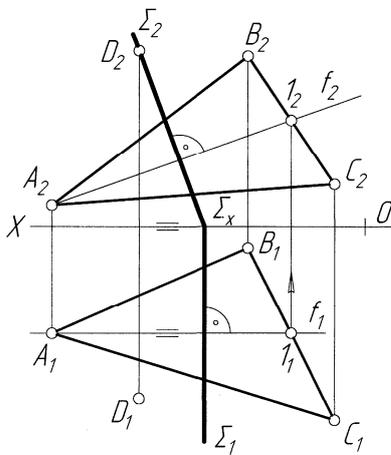


Figure 4.40

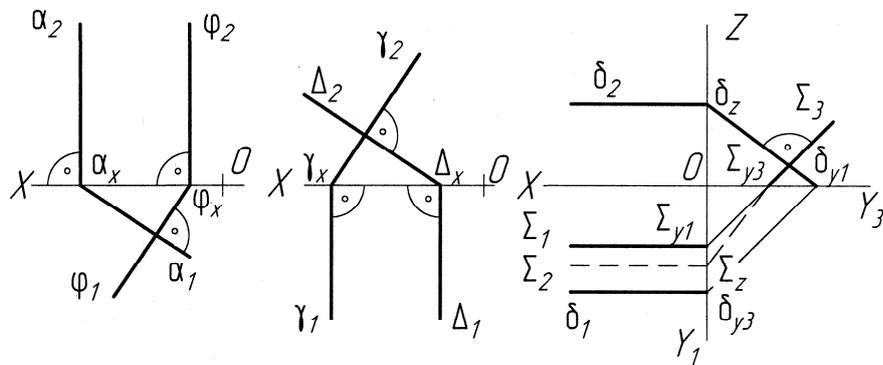


Figure 4.41

5 CONVERSION OF PROJECTION

5.1 CHANGE OF PROJECTION PLANES

The introduction of auxiliary projection planes provides another method of attack on a problem. A general case may often be reduced to a special case having a simple solution by choosing suitable auxiliary views. The auxiliary-view (auxiliary-projection) method of solution is especially useful when the given data lies in an unfavorable position with respect to the projection planes. For example, the true distance between two parallel straight lines which are inclined to the horizontal and frontal projection plane can be measured readily in the auxiliary view in which the lines project as points.

5.2 CHANGE OF PROJECTION PLANES (SECONDARY PLANES OF PROJECTION)

In practical work, views of objects are often wanted in other directions than those which are obtained by the use of the horizontal and frontal coordinate planes. In fact, it often happens that an actual object cannot be adequately represented by a simple plan and elevation. In the theory, also, an additional projection in a suitably chosen direction may give readily a solution otherwise difficult to obtain. Such views, or projections, are obtained on secondary planes of projection, taken perpendicular to either Π_1 or Π_2 , and making any angle whatever with the other coordinate planes.

Secondary planes of projection are by preference taken perpendicular to Π_1 , for while any plane perpendicular to Π_1 is vertical and named secondary frontal plane of projection (Π_4, Π_6, \dots), a plane perpendicular to Π_2 is neither vertical nor horizontal, and therefore does not conform to either of the natural directions. This plane is named secondary horizontal plane of projection (Π_5, Π_7, \dots).

A very important secondary plane of projection is profile plane, which is perpendicular to both Π_1 and Π_2 . This plane has just been discussed in detail in the preceding lecture. Indeed, this plane can be considered as a third coordinate plane of equal rank with the horizontal and frontal coordinate planes. The profile plane can be considered as a special case of a plane perpendicular to Π_2 .

The method of projecting on a secondary frontal plane is shown in Fig. 5.1. The secondary plane Π_4 intersects Π_1 in a secondary axis, x_{14} (subscripts are similar to plane subscripts), which may be at any angle with the original axis x_{12} . Since the projectors $A_2A_{x_{12}}$ and $A_4A_{x_{14}}$ are both parallel to Π_1 , the distances $A_2A_{x_{12}}$ and $A_4A_{x_{14}}$ are equal, both being equal to AA_1 , the distance of the point from Π_1 .

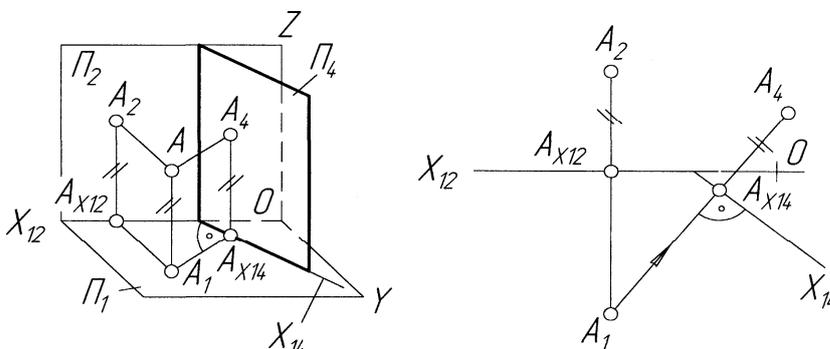


Figure 5.1

The method of projecting on a secondary horizontal plane is shown in Fig. 5.2. The secondary plane Π_5 intersects Π_2 in a secondary axis, x_{25} , which may be at any angle with the original axis x_{12} . Since the projectors $A_1A_{x_{12}}$ and $A_5A_{x_{25}}$ are both parallel to Π_2 , the distances $A_1A_{x_{12}}$ and $A_5A_{x_{25}}$ are equal, both being equal to AA_2 , the distance of the point from Π_2 .

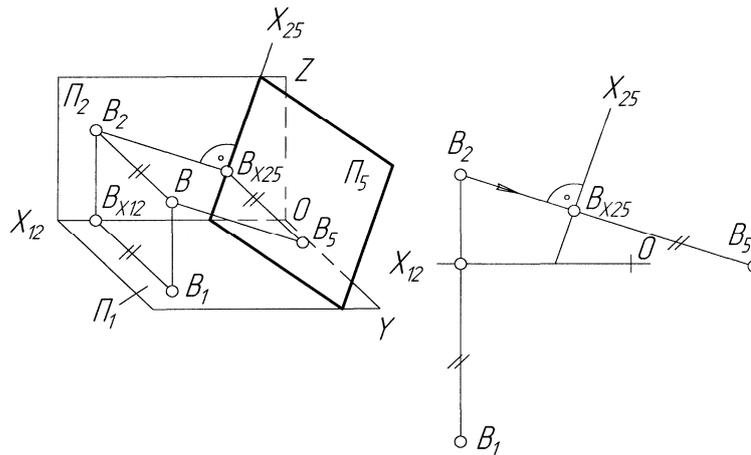


Figure 5.2

5.3 PRINCIPLES OF SECONDARY PROJECTIONS

In general for any secondary frontal plane of projection we have the following propositions:

- The plane may be viewed from either side, irrespective of the position of the object, but the direction of sight must always be at right angles to the plane, and the projectors must be drawn at right angles to x_{14} .
- Distances above Π_1 must be laid off on the farther side of x_{14} , and distances below Π_1 on the near side.
- All vertical distances, either above or below Π_1 , remain unchanged.

For any secondary horizontal plane of projection we have the similar propositions.

Simplification of problems by means of secondary projections

In the solution of problem, there is no advantage in introducing a secondary plane of projection unless the new projection is in some way simpler than the original projections.

A point always projects as a point, and cannot be made any simpler.

The simplest projection of a straight line is a point. This projection can be obtained by a single secondary plane of projection only when the given line is parallel to one of the original coordinate planes. Let the line AB be parallel to Π_1 (Fig. 5.3); then if x_{14} is taken perpendicular to A_1B_1 , the line AB will be seen endwise, and the projection A_4B_4 will become a point.

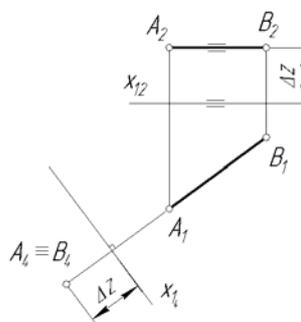


Figure 5.3

Another simple and useful projection of a straight line results when the line is parallel to a plane of projection. This can always be attained by means of a secondary Π_4 , by taking x_{14} parallel to the Π_1 -projection of the line, as shown in Fig. 5.4.

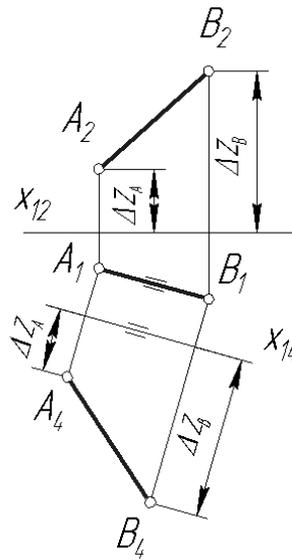


Figure 5.4

Thus, transformation of inclined straight line to projecting position can always be attained by means of two steps: the first - by taking first secondary axis x parallel to the one of projection of the line (for example, x_{14} parallel A_1B_1 as shown in Fig. 5.5), the second - by taking second secondary axis x perpendicular to the new projection of the line (x_{45} perpendicular to A_4B_4) (Fig. 5.5).

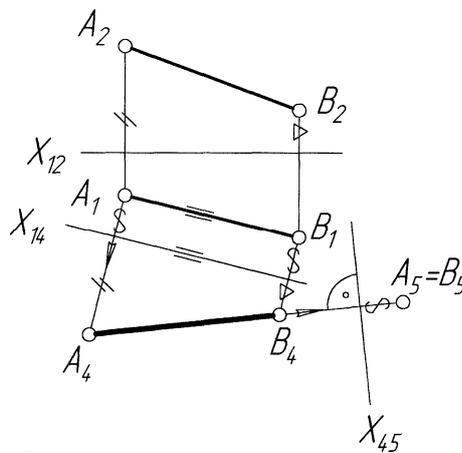


Figure 5.5

The simplest position of a plane is that in which one trace is an edge view of the plane. Such a view can always be obtained by means of a secondary frontal plane of projection. Let φ (Fig. 5.6), be a general plane. Assume a secondary axis x_{14} perpendicular to φ_1 . Then, since φ_1 is perpendicular to this axis, φ is perpendicular to Π_4 , and its trace, φ_4 , on the secondary frontal plane Π_4 is an edge view of φ . To find φ_4 : since φ is seen edgewise against Π_4 , φ_4 will pass through the projection of any point in φ .

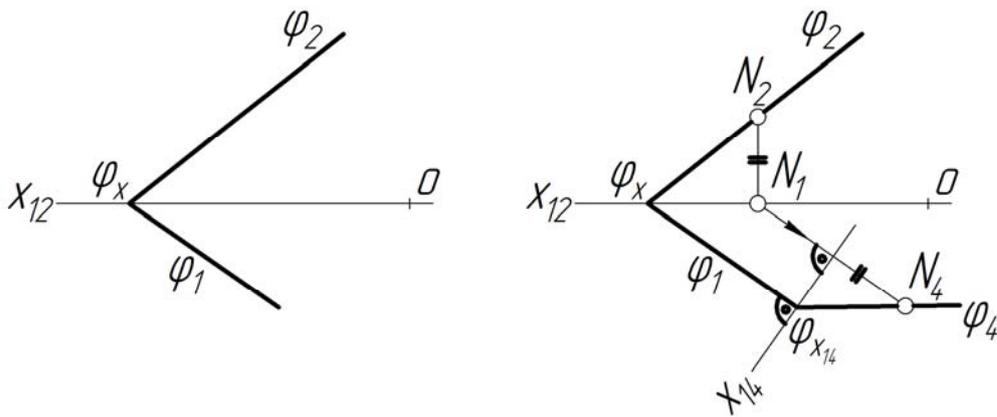


Figure 5.6

Let plane is given by another way (for example by triangle ABC , as shown in Fig. 5.7). In this case assume a secondary axis x_{14} perpendicular to h_1 (h_1 is parallel to the horizontal trace of the plane ABC). Then, since h_1 is perpendicular to this axis, ABC is perpendicular to Π_4 , and its projection, $A_4B_4C_4$, on the secondary frontal plane Π_4 is an edge view of ABC .

Another simple projection of a plane results when the plane is parallel to a plane of projection. This can always be attained by means of a next secondary plane of projection, by taking secondary axis x parallel to the obtain projection of the plane, as shown in Fig. 5.7.

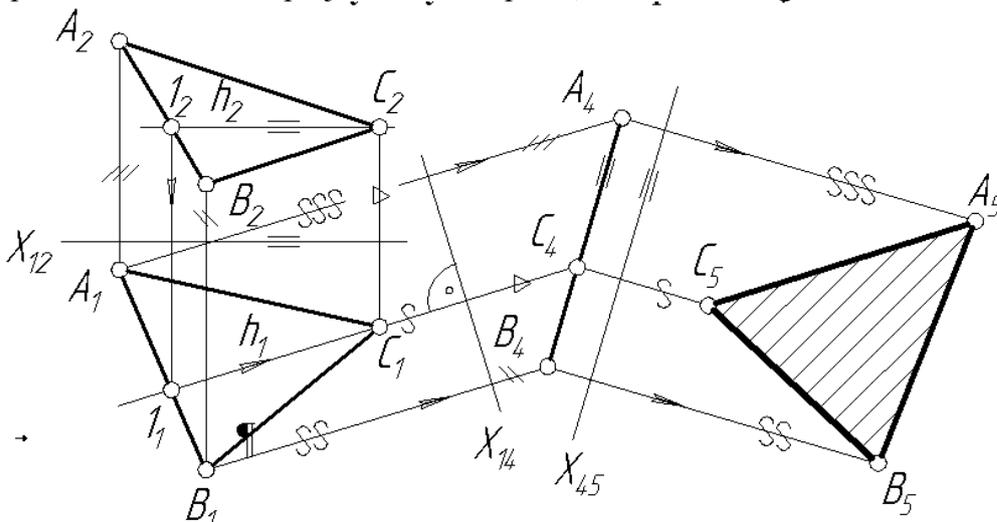


Figure 5.7

Thus, transformation of inclined plane to level position can always be attained by means of two steps: the first - by taking first secondary axis x perpendicular to the one trace of the plane or its principal line of the same name (for example, x_{14} is perpendicular h_1 as shown in Fig. 5.7), the second - by taking second secondary axis x parallel to the new projection of the line (x_{45} parallel to $A_4B_4C_4$) (Fig. 5.7).

5.4 THE METHOD OF ROTATION

An alternate method for determining normal views of oblique lines or planes- is to rotate the line or plane into parallelism with one of the projection planes.

The fundamental construction of the method of rotation consists in rotating a space point about an axis which is perpendicular plane of projection. A point rotated about an axis moves at a constant distance from the axis. The path is a circle whose plane is perpendicular to the axis.

The rotation of a point about a vertical (horizontal-projecting) axis

As an illustration of the general case of this problem, let consider the rotation of a point about a vertical (horizontal-projecting) axis (Fig. 5.8).

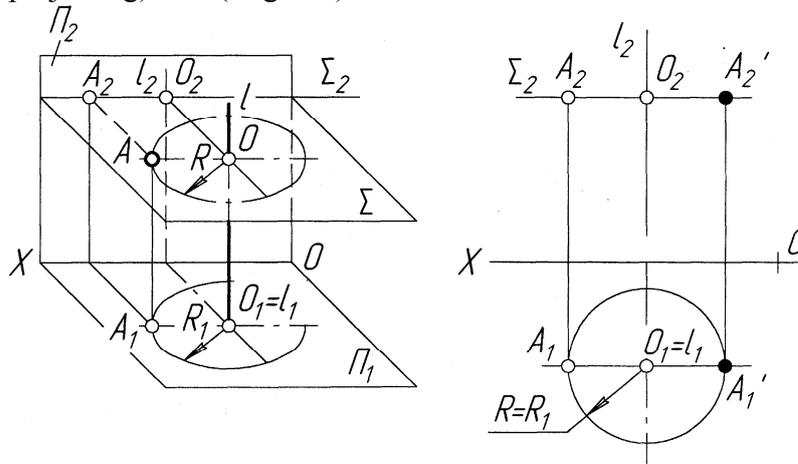


Figure 5.8

Five elements of the rotation:

- object of the rotation – point $A(A_1, A_2)$;
- axis of the rotation – line $l(l_1, l_2)$, perpendicular to the plane of projection (to Π_1 in Fig. 5.8);
- plane of the rotation – plane $\Sigma(\Sigma_1, \Sigma_2)$, which is perpendicular to axis l of the rotation and consists object of the rotation (point A);
- center of rotation – point $O(O_1, O_2)$, which is point of intersection of the plane Σ of the rotation and axis of the rotation l ;
- radius of the rotation - radius of the circle which is passed by object of the rotation (point A) and is equal to distance from the axis of the rotation l to object of the rotation A .

To find the true length of a straight line segment

An oblique line projects in its true length when rotated into parallelism with the vertical or horizontal plane about an axis through one point of the line (Fig. 5.9).

Rotate point B about a frontal projecting axis through A until AB becomes parallel to the horizontal plane of projection. The revolved horizontal view ($A_1B'_1$) is the true-length view. The angle β is the true angle between AB and the frontal plane of projection. This construction for true length is usually simplified by omitting the revolved front view and setting off the distance OB' equal to A_2B_2 .

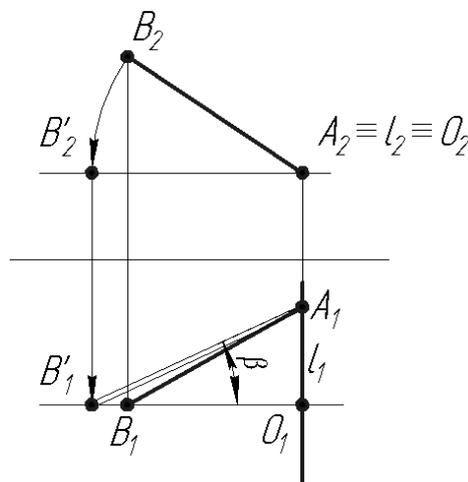


Figure 5.9

The true length of line AB may also be obtained by assuming the axis through point A and perpendicular to the horizontal plane of projection (Fig. 5.10). Line AB is rotated parallel to the frontal plane of projection and projects in true length in the front view. The angle α is the true angle between AB and the horizontal plane of projection.

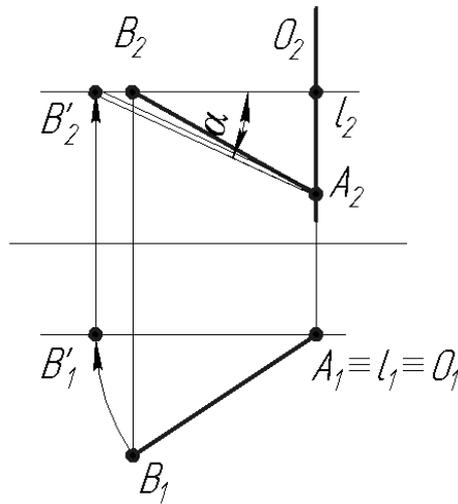


Figure 5.10

A straight line, when revolved about an axis perpendicular to Π_1 , maintains always its original angle with Π_1 . If the axis is perpendicular to Π_2 , the angle between the revolving line and Π_2 does not change.

As in the revolution of a straight line, a plane, when revolved about an axis perpendicular to Π_1 , maintains always its original angle with Π_1 . If the axis is perpendicular to Π_2 , the angle between the revolving plane and Π_2 does not change (Fig. 5.11).

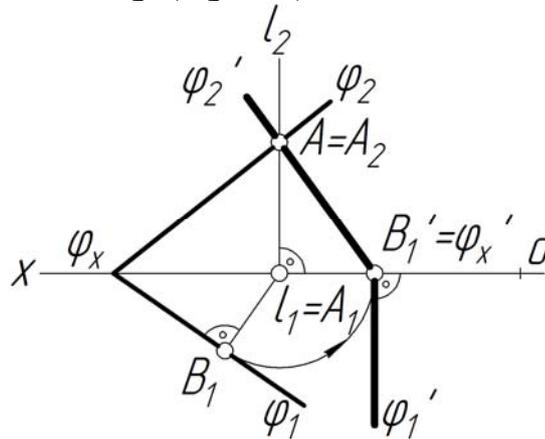


Figure 5.11

A frontal-projecting plane (ABC) projects in its true shape when rotated into parallelism about a frontal-projecting axis l through one point of the plane (point A) (Fig. 5.12).

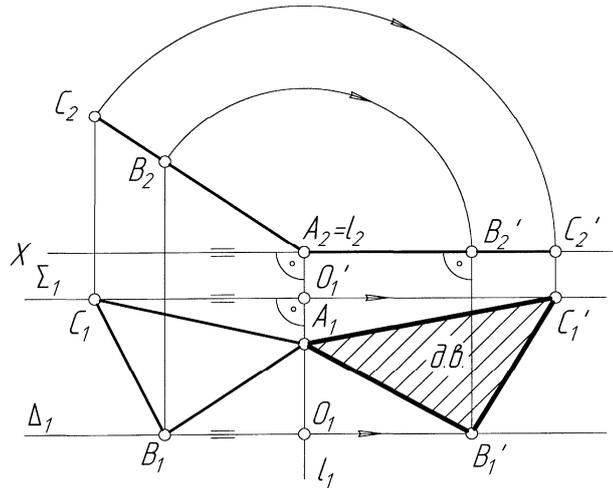


Figure 5.12

5.5 ROTATION ABOUT THE PRINCIPAL LINES OF A PLANE

Rotation of a point about a horizontal axis (Fig. 5.13). Point C and the horizontal line h are given. To rotate C about h as an axis until it lies in the horizontal plane which contains h . Plane Σ of rotation of point C is perpendicular to h (Σ_1 is perpendicular to h_1). The path of rotation is a circle. In the horizontal plane of projection (top view), this circle projects as a straight line perpendicular to h_1 , with its center at O . The circle projects in true shape in an auxiliary view in which the axis h projects as a point. In space, point C rotating about h describes a circle of radius R_{rot} . The true length of R_{rot} may also be obtained by right-angled triangle. Point C lies in the horizontal plane containing h at the points in which horizontal plane cuts the circle. The top views of these two points are C_0 and C'_0 .

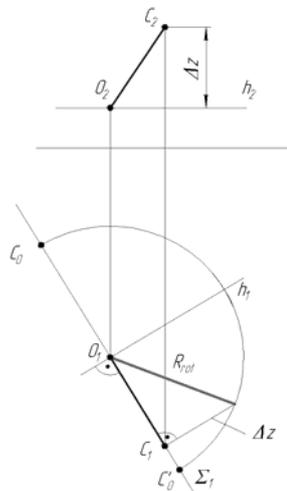


Figure 5.13

A plane figure appears in its true shape when rotated into or parallel to a projection plane. The axis about which the figure is rotated must be contained in the plane and be either in the projection plane or parallel to it.

To find the true shape of the triangle ABC

Draw the projections of the horizontal line h of the plane ABC . Take h_2 through A_2I_2 , construct h_1 . Rotate points B and C about h as an axis by the method of Figure 5.13, obtaining B_0 and C_0 . The rotated view $A_0B_0C_0$ is the true shape of ABC (Fig. 5.14).

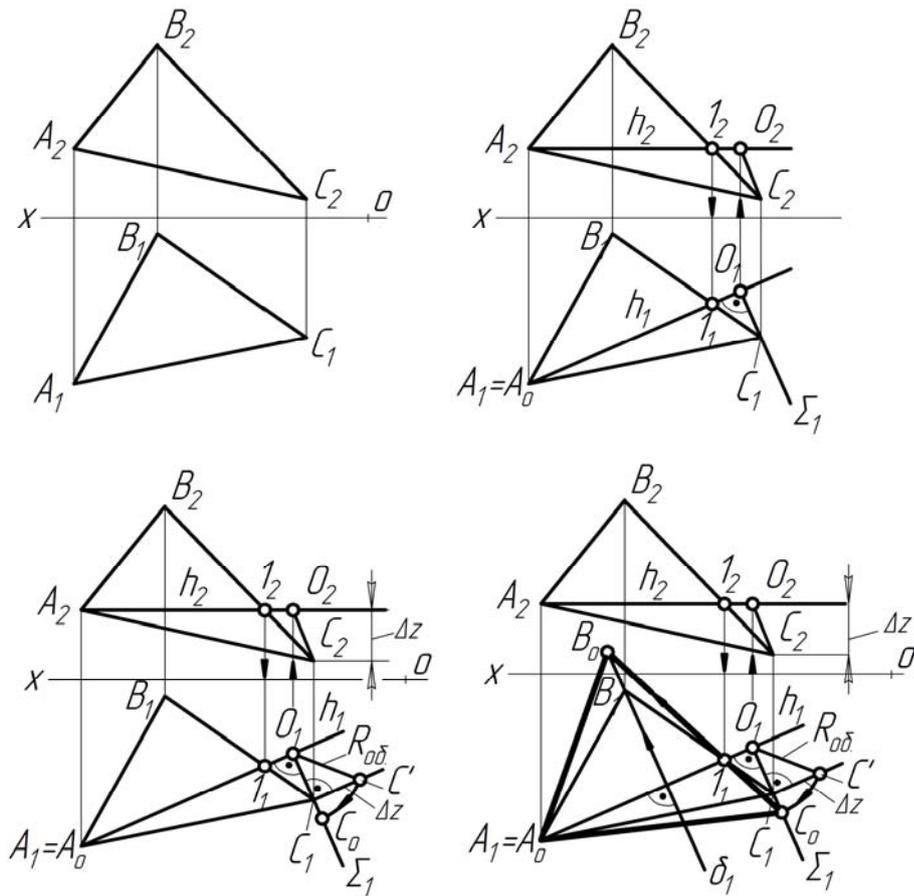


Figure 5.14

5.6 METHOD OF SUPERPOSITION

Traces are particular case of the principal lines of a plane, therefore this method is particular case of previous method (rotation about the principal lines of a plane).

This method is superposition of given plane on horizontal or frontal plane of projection by rotation about a horizontal or frontal trace of given plane respectively (Fig. 5.15). Plane $\alpha(\alpha_1, \alpha_2)$ is given. To rotate any point $N(N_1, N_2)$ of frontal trace about horizontal trace α_1 as an axis until it lies in the horizontal plane of projection Π_1 . Plane δ of rotation of point N is perpendicular to horizontal trace (δ_1 is perpendicular to α_1). The path of rotation is a circle. In the horizontal plane of projection (top view), this circle projects as a straight line perpendicular to α_1 , with its center at $O(O_1, O_2)$. Point N_0 lies in the horizontal plane of projection at the point in which the circle cuts Π_1 . Distance from N_0 to α_x and distance from α_x to N_2 are equal (in space and on sketch).

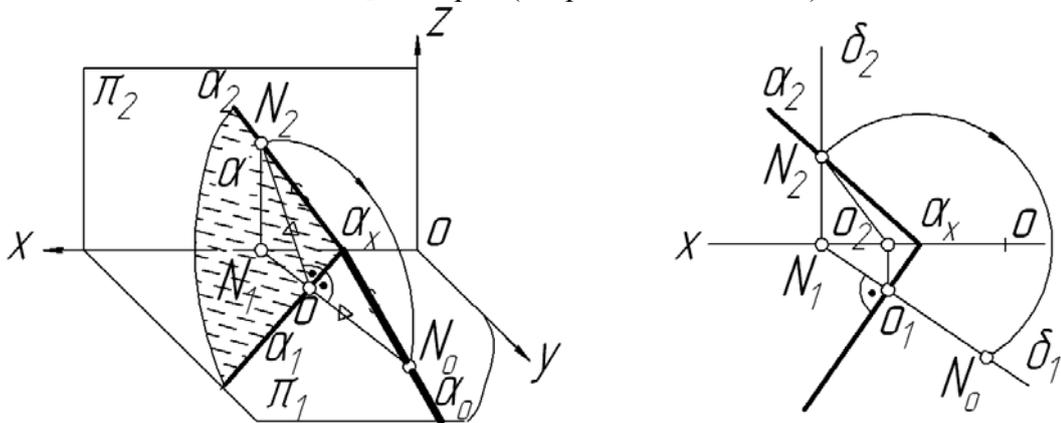


Figure 5.15

To find the true shape of triangle ABC.

The triangle is to be rotated into the Π_1 -plane about its trace on the Π_1 -plane in way as stated above. Point A as the point of frontal trace lies in the horizontal plane of projection at the point A_0 in which distance from A_0 to Σ_x and distance from Σ_x to A_2 are equal (in space and on sketch). Point C as the point of horizontal trace (axis of rotation) lies in the horizontal plane of projection at the same point ($C_1 \equiv C_0$).

Point B lies on horizontal line h of plane Σ ($C_1 \in h_1$, $C_2 \in h_2$). h_0 is constructed using

superposed frontal trace N_0 of h and h_0 and Σ_1 parallelism. Plane of rotation of point B is perpendicular to horizontal trace Σ_1 . B_0 lies in the horizontal plane of projection at the point in which plane of rotation intersects h_0 .

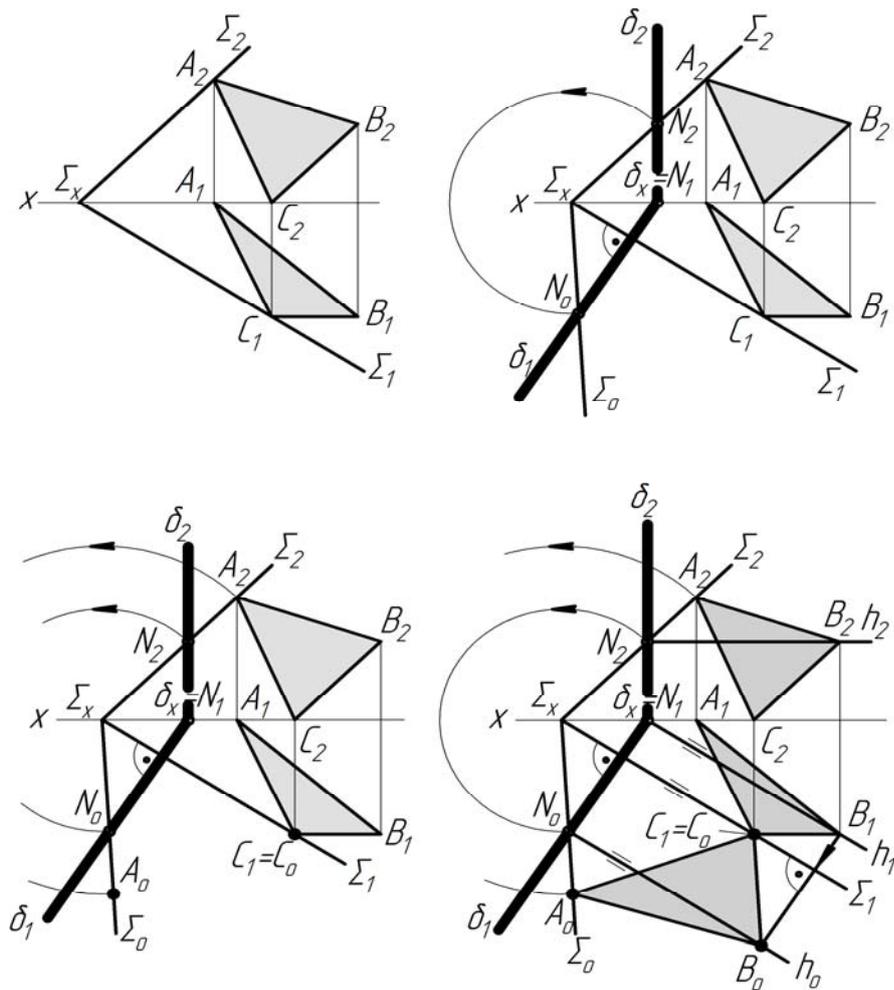


Figure 5.16

6 SURFACES AND SOLIDS

6.1 POLYHEDRONS

A **polyhedron** is solid that is bounded by polygons, called **faces** that enclose a single region of space. An **edge** of polyhedron is a line segment formed by the intersection of two faces. A **vertex** of a polyhedron is a point where there or more edges meet. The plural of polyhedron is **polyhedra**, or polyhedrons (Fig. 6.1).

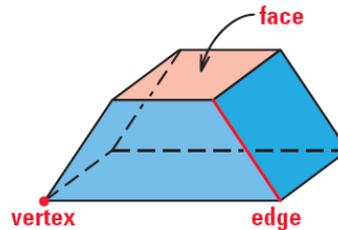


Figure 6.1

A polyhedron is **regular** if all of its faces are congruent regular polygons. A polyhedron is **convex** if any two points on its surface can be connected by a segment that lies entirely inside or on the polyhedron (Fig. 6.2, a). The polyhedron is **nonconvex**, or **concave** if this segment goes outside the polyhedron (Fig. 6.2, b).



Figure 6.2

There are five regular polyhedra, called **Platonic solids**, after the Greek mathematician and philosopher Plato. The **Platonic solids** are regular **tetrahedron** (4 faces), a **hexahedron** or cube (6 faces), a regular **octahedron** (8 faces), a regular **dodecahedron** (12 faces), and a regular **icosahedron** (20 faces) (Fig. 6.3).

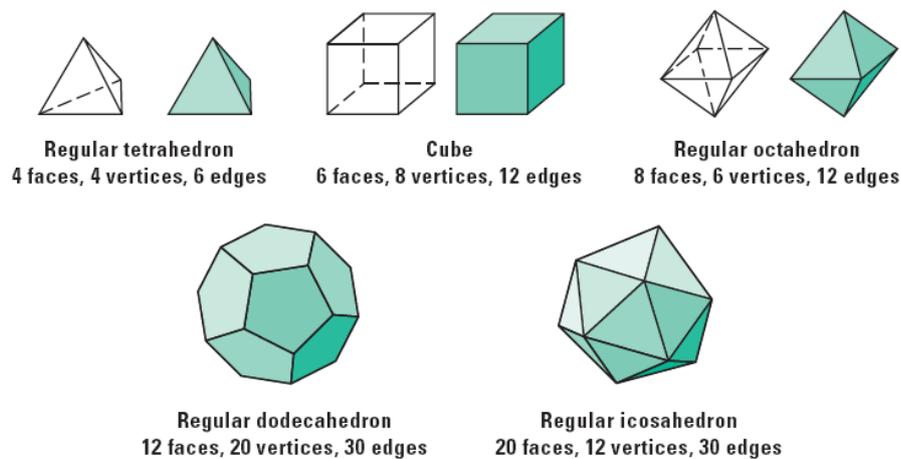


Figure 6.3

Platonic solids end in “hedron”. Hedron is Greek for “side” or “face”.

The sum of the number of faces and vertices is two more than the number of edges in the solids above. This result was proved by the Swiss mathematician Leonhard Euler (1707 - 1783).

Euler's Theorem: The number of faces (F), vertices (V), and edges (E) of a polyhedron are related by the formula:

$$F + V = E + 2.$$

Prism: a solid bounded on the sides by parallelograms and on the ends by polygonal figures in parallel planes.

Right prism is a prism in which the joining edges and faces are perpendicular to the base faces. This applies if the joining faces are rectangular. If the joining edges and faces are not perpendicular to the base faces, it is called an *oblique prism*.

Regular right prism - a right prism with a **base** that is a **regular polygon** (Fig. 6.4).

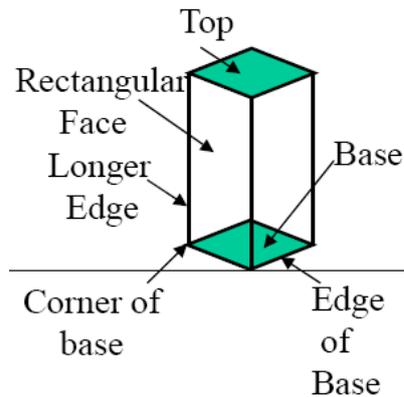


Figure 6.4

Pyramid is a solid bounded on the sides by triangles and on the ends by polygonal figure.

Right pyramid is a pyramid that has its apex aligned directly above the center of the base.

Regular right pyramid - a right pyramid with a **base** that is a **regular polygon** (Fig. 6.5).

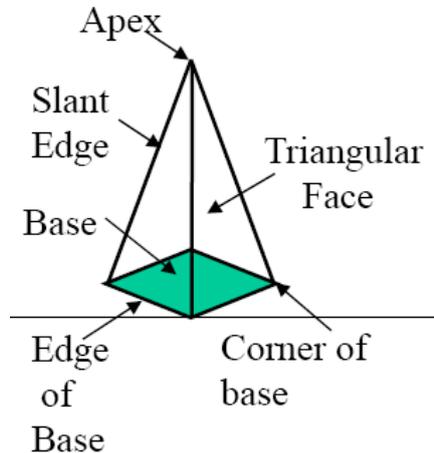


Figure 6.5

Truncated pyramid: Section of solid (top and base not parallel) (Fig. 6.6).

Frustum of pyramid: Top and base are parallel to each other (Fig. 6.7).

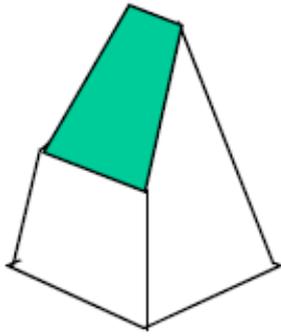


Figure 6.6

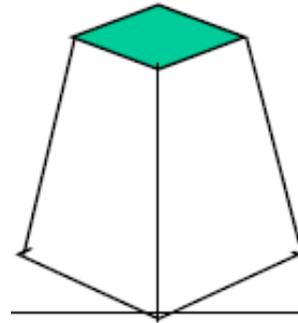


Figure 6.7

6.2 POINTS AND LINE ON THE SURFACE OF THE SOLID

The point lies on a surface if lies on a line lying on this surface (Fig. 6.8).

The line lies in a face if its two points lie in this face (Fig. 6.8).

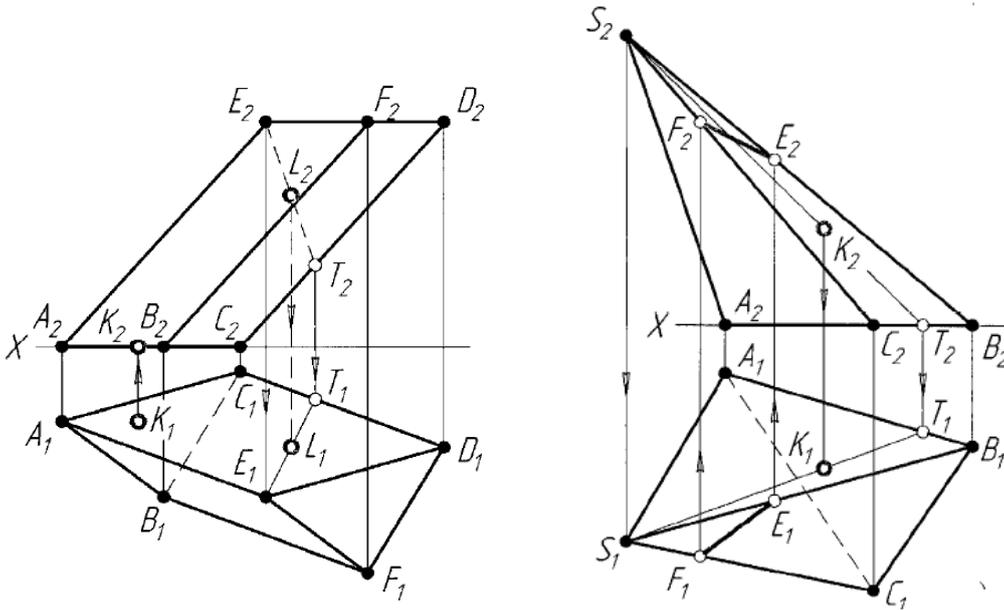


Figure 6.8

Auxiliary intersection

It may be proved by geometry that if any prism, pyramid is cut by a plane parallel to the base, the intersection is always a figure like the base in shape, and parallel to it in position (Fig. 6.9).

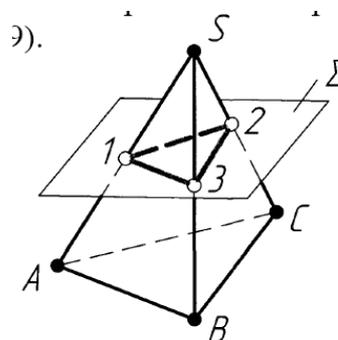


Figure 6.9

Auxiliary intersection is often used for face point finding.

Let α represent a plane seen edgewise (Fig. 6.10). Then α is parallel to the Π_1 plane, and perpendicular to Π_2 . This plane cutting the pyramid will intersect all of the inclined faces, giving thereby three lines of intersection. The plane α , seen edgewise in elevation, cuts the slanting edges in points 1 , 2 , and 3 ; and these points are found in plan directly below on the corresponding edges.

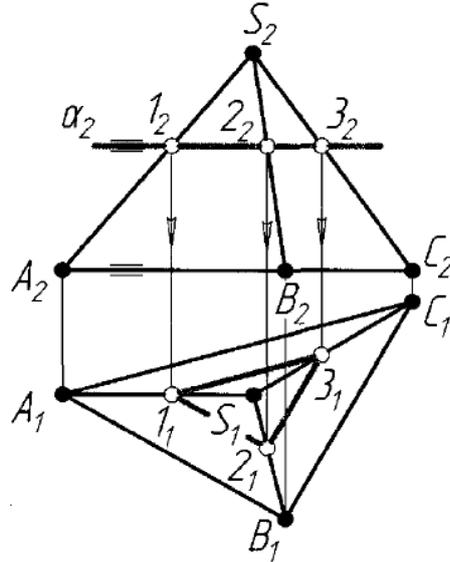


Figure 6.10

As the plane cuts edge AS at 1 , and CS at 3 , it will cut the face SAC in the line joining 1 and 3 ; and, by joining point 2 with 1 and 3 , the other two lines are determined, and the whole intersection 123 completed.

The sides of the triangle 123 are respectively parallel to the edges of the base of the pyramid. This parallel relation is due to the fact that the plane α in this case is parallel to the base.

Auxiliary intersection is often used for edge point finding, for example projection of point D (Fig. 6.11).

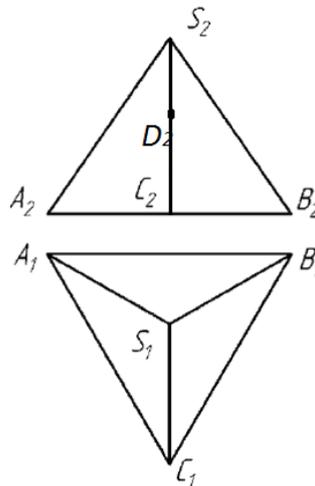


Figure 6.11

6.3 SURFACES OF REVOLUTION

A **surface of revolution** is a surface created by rotating a curve (the generatrix or the generator) around a straight line (the axis) (Fig. 6.12).

Examples of surfaces generated by a straight line are cylindrical (Fig. 6.13) and conical surfaces (Fig. 6.14) when the line is coplanar with the axis, as well as hyperboloids of one sheet when the line is skew to the axis (Fig. 6.15).

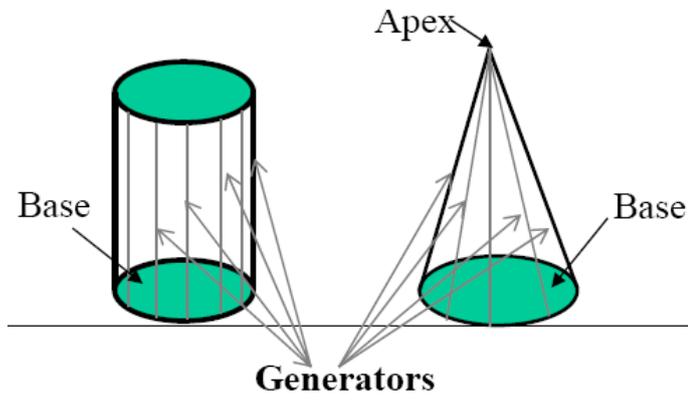


Figure 6.12

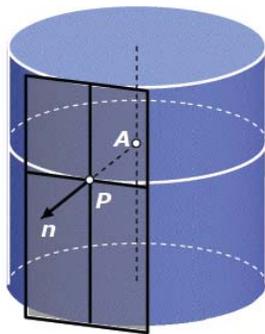


Figure 6.13

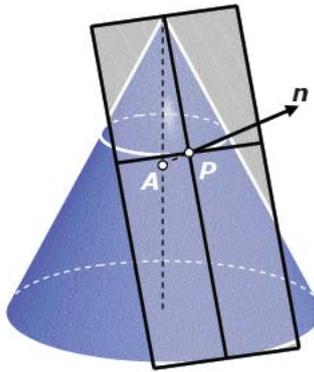


Figure 6.14

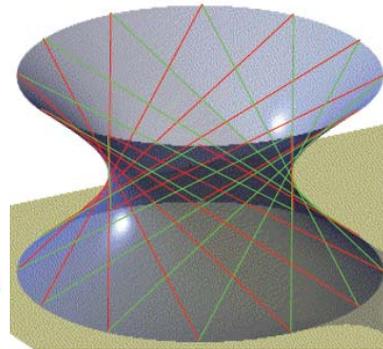


Figure 6.15

A circle that is rotated about a diameter generates a **sphere** (Fig. 6.16) and if the circle is rotated about a coplanar axis other than the diameter it generates a **torus** (Fig. 6.17).

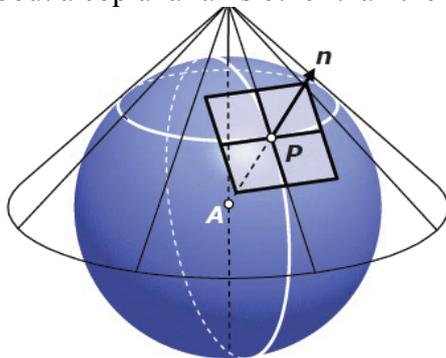


Figure 6.16

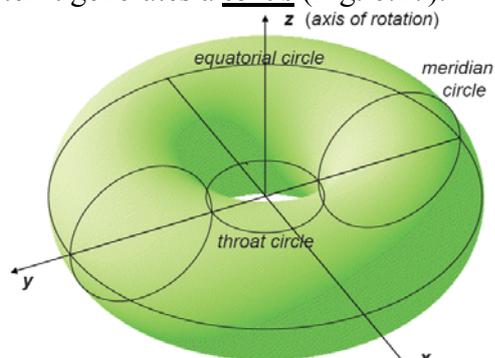


Figure 6.17

Truncated solids: Section of solids (top and base not parallel) (Fig. 6.18).

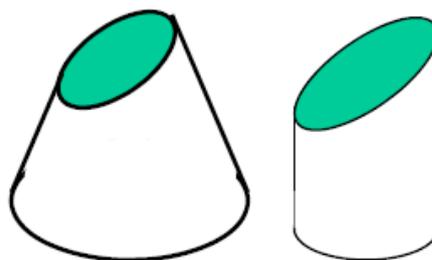


Figure 6.18

Frustum of: Top and base are parallel to each other (Fig. 6.19).

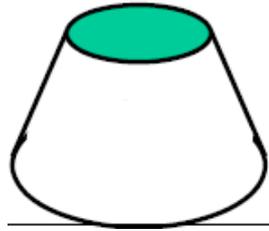


Figure 6.19

6.4 POINTS ON THE SURFACES

The point lies on a surface if lies on a **line lying** on this surface (Fig. 6.20, 6.21, 6.22).

It is well to remind that if any prism, pyramid, cylinder, or cone is cut by a plane parallel to the base, the intersection is always a figure like the base in shape, and parallel to it in position.

It is well also to remind that if any surface of revolution is cut by a plane perpendicular to the axis, the intersection is always a circle.

Auxiliary intersection is parallel and congruent to the base. It is often used for surface point finding (Fig. 6.22). Generator method is used for surface point finding too (Fig. 6.21).

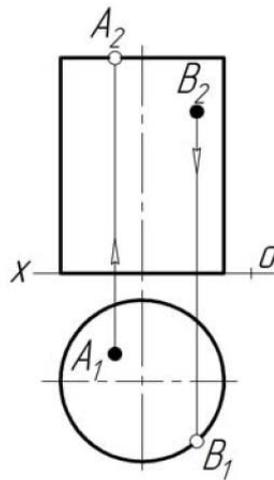


Figure 6.20

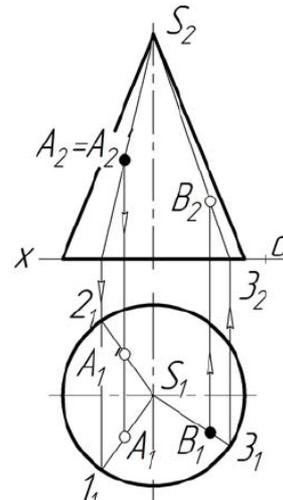


Figure 6.21

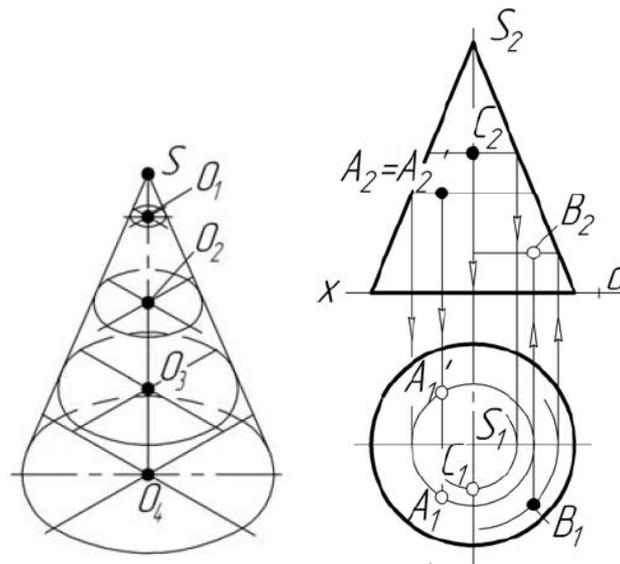


Figure 6.22

6.5 DEVELOPMENTS

- A development is the unfolded or unrolled, flat or plane figure of a 3-D object.
- Called a pattern, the plane figure may show the true size of each area of the object. When the pattern is cut, it can be rolled or folded back into the original object.
- A true development is one in which no stretching or distortion of the surface occurs, and every surface of the development is the same size and shape as the corresponding surface on the 3-D object.
- Only polyhedrons and single-curved surfaces can produce true developments.

By the term development is meant a figure showing the true size and shape of the surface of any given solid, when the surface has been rolled or spread out on a plane. A true development is obtained when all parts of the surface is made to coincide with a plane, without overlapping or folding. Such a development, when cut out of paper or thin sheet metal, and properly bent or rolled up, will reproduce the original solid.

Some surfaces are truly developable, and others are not. Among the former are all surfaces composed of planes, as those of prisms and pyramids, all cylindrical, and all conical surfaces. The most familiar solid which has a non-developable surface is the sphere.

6.5.1 METHODS TO DEVELOP SURFACES

1. Parallel-line development: Prismatic objects (cylinder, prism)
2. Radial-line development: Non-prismatic objects (cone, pyramid). Apex as center and slant edge as radius.
3. Triangulation development: Complex shapes are divided into a number of triangles and transferred into the development
4. Approximation.

All lines are **TRUE LENGTH**

All planar surfaces are **TRUE SIZE**

Types of Developments

Parallel-line (Fig. 6.23)

Radial-line (Fig. 6.24)

Triangulation (Fig. 6.25)

Approximation (Fig. 6.26)

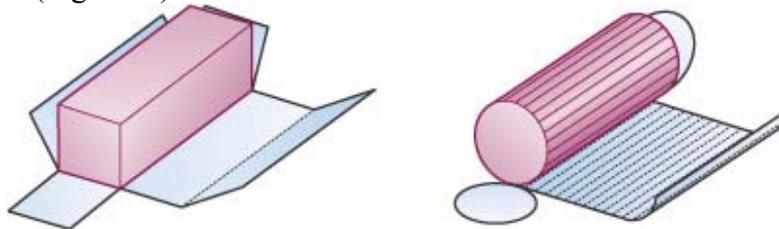


Figure 6.23



Figure 6.24

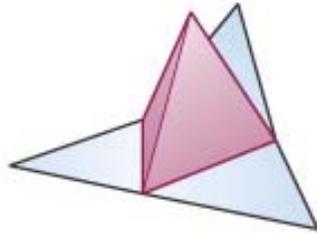


Figure 6.25

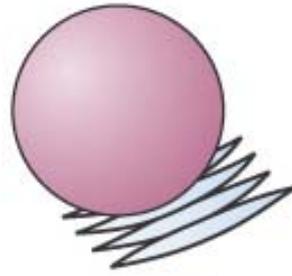


Figure 6.26

Parallel-line

Rectangular Prism. In order to find the development of the rectangular prism in Fig. 6.27, the back face, $B'-C'-C-B$, is supposed to be placed in contact with some plane, then the prism turned on the edge $C'-C$ until the side $C'-D'-D-C$ is in contact with the same plane, and this process continued until all four faces have been placed on the same plane. Rectangles $B'-C'-D'-A'$ and $B-C-D-A$ are for the top and bottom respectively. The development then is the exact size and shape of a covering for the prism. If a rectangular hole is cut through the prism, the openings in the front and back faces will be shown in the development in the centers of the two broad faces.

The development of a right prism, then, consists of as many rectangles joined together as the prism has sides, these rectangles being the exact size of the faces of the prism, and in addition two polygons the exact size of the bases. It will be found helpful in developing a solid to number or letter all of the corners on the projections, then designate each face when developed in the same way as in the figure.

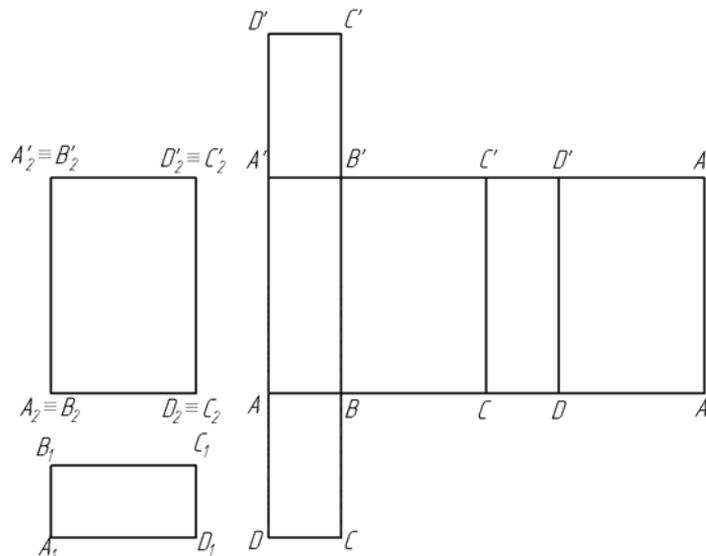


Figure 6.27 Development of Rectangular Prism.

Circular Cylinder. If a circular cylinder is to be developed, or rolled upon a plane, the elements, being parallel, will appear as parallel lines, and the base line being perpendicular to the elements, will appear as a straight line of length equal to the circumference of the base. The base of the cylinder in Fig. 6.28 is divided into twelve equal parts, I, II, III, etc., and commencing at point I on the development, the twelve equal spaces are laid off along the straight line, giving the total width.

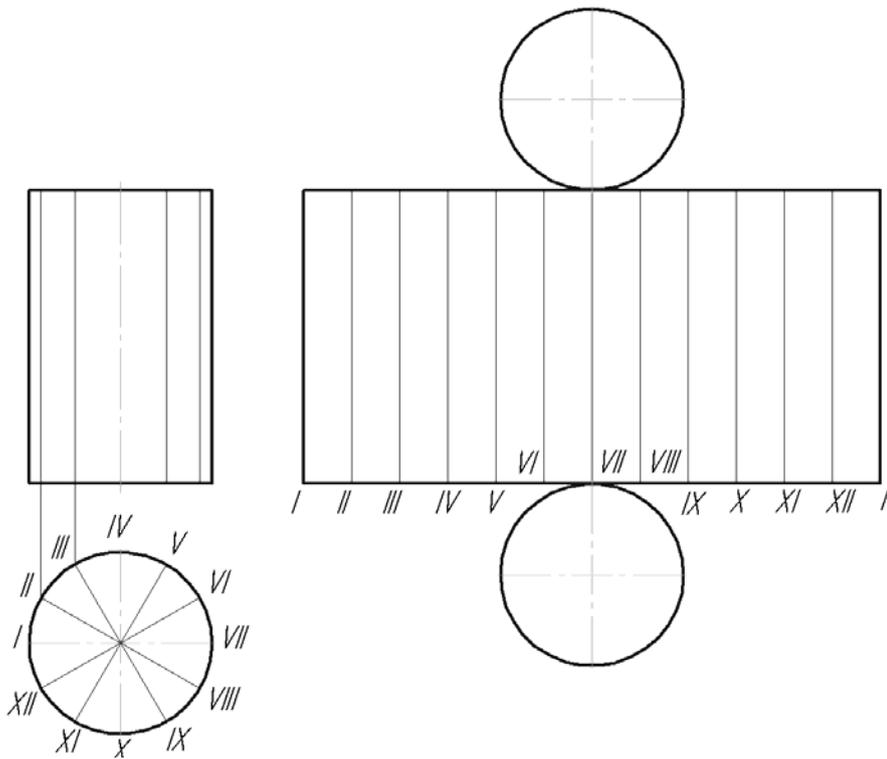


Figure 6.28

Radial-line

Regular Triangular Pyramid. Fig. 6.29 represents the plan and elevation of a regular triangular pyramid, and its development. If face ABC is placed on the plane, its true size will be shown in the development. The true length of the base of triangle ABC is shown in the plan. As the slanting edges, however, are not parallel to the vertical, their true length is not shown in elevation but must be obtained by the method of rotation (for example), as indicated in Fig. 88. The triangle may now be drawn in its full size at SAC in the development, and as the pyramid is regular, two other equal triangles, SCB and SAB , may be drawn to represent the other sides. These, together with the base ABC , constitute the complete development.

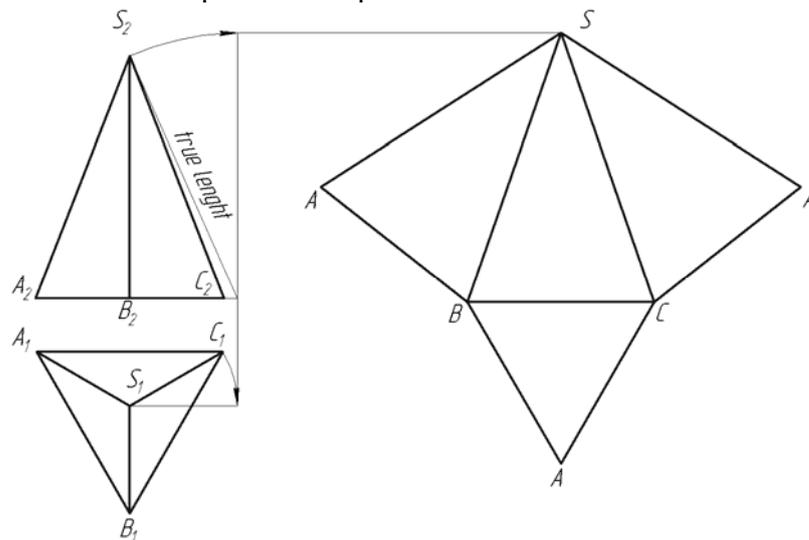


Figure 6.29 Plan, Elevation of Triangular Pyramid and Development of Triangular Pyramid.

Cone. If a cone be placed on its side on a plane surface, one element will rest on the surface. If now the cone be rolled on the plane, the vertex remaining stationary until the same element is in contact again, the space rolled over will represent the development of the convex surface of the

cone. In Fig. 6.30, let the vertex of the cone be placed at Π_1 , and one element of the cone coincide with plane. The length of this element is taken from the frontal projection, of either contour element. All of the elements of the cone are of the same length, so that when the cone is rolled, each point of the base as it touches the plane will be at the same distance from the vertex. From this it follows that in the development of the base, the circumference will become the arc of a circle of radius equal to the length of an element, and of a length equal to the distance around the base. To find this length divide the circumference of the base in the plan into any number of equal parts, say twelve, and lay off twelve such spaces I, II, III, etc. along an arc drawn with radius equal to S_2I_2 (S_2VII_2); join I and I with S, and the resulting sector is the development of the cone from vertex to base.

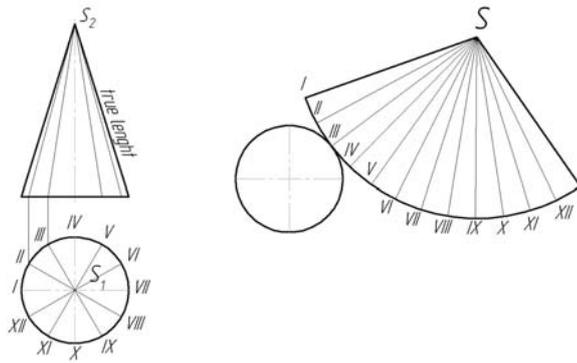


Figure 6.30 Plan, Elevation of Cone and Development of Cone.

Approximate Development

An approximate development is one in which stretching or distortion occurs in the process of creating the development.

The resulting flat surfaces are not the same size and shape as the corresponding surfaces on the 3-D object.

Warped surfaces do not produce true developments, because pairs of consecutive straight-line elements do not form a plane.

Double-curved surfaces such as spheres do not produce true developments.

There are two methods for construction of sphere development.

Zone Method: Development of sphere using frustum of cones (polyconic). Fig. 6.31 represents half of sphere development.

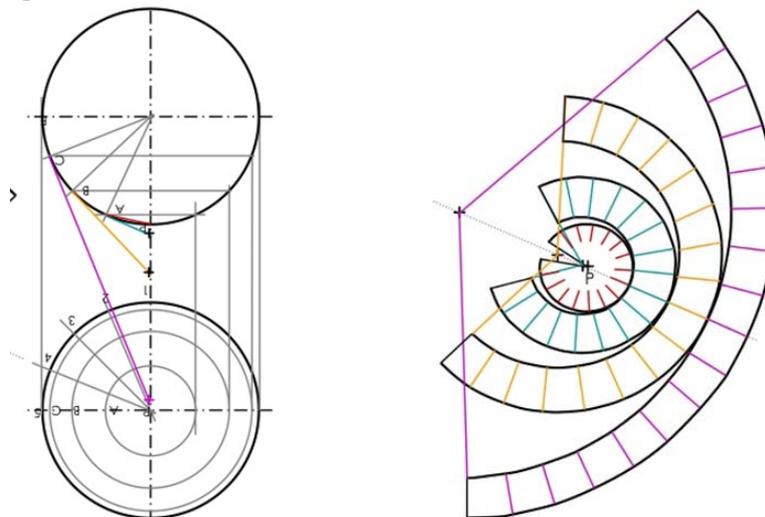


Figure 6.31

Polycylindric or Gore Method: Development of sphere using lunes. Fig. 6.32 represents half of hemisphere development.

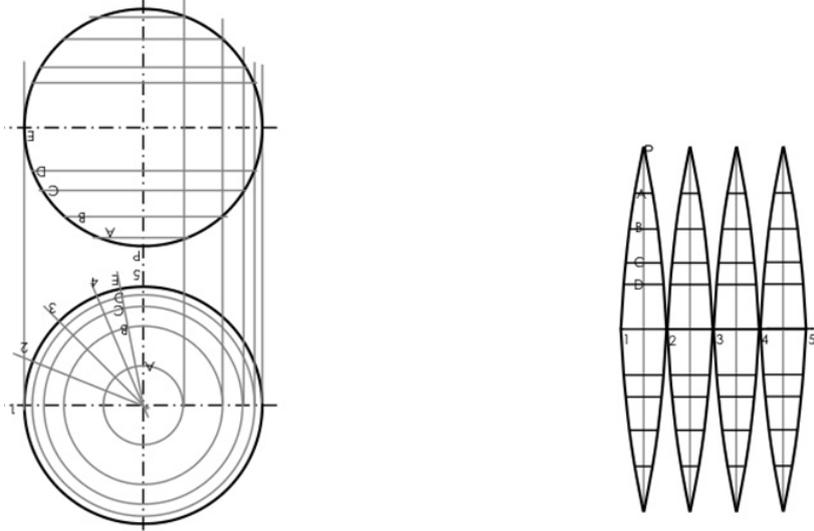


Figure 6.32

7 INTERSECTIONS OF SOLIDS AND SURFACES

Line method: A number of lines are drawn on the lateral surface of one of the solids and in the region of the line of intersection. Points of intersection of these lines with the surface of the other solid are then located. These points will lie on the required line of intersection. They are more easily located from the view in which the lateral surface of the second solid appears edgewise (i.e. as a line). The curve drawn through these points will be the line of intersection.

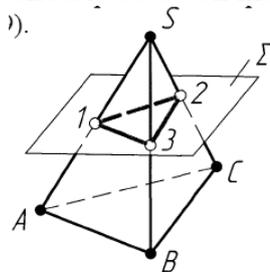
Cutting-plane method: The two solids are assumed to be cut by a series of cutting planes. The cutting planes may be vertical (i.e. perpendicular to the horizontal plane Π_1), edgewise (i.e. perpendicular to the frontal plane Π_2) or oblique. The cutting planes are so selected as to cut the surface of one of the solids in straight lines and that of the other in straight lines or circles.

7.1 INTERSECTIONS OF POLYHEDRON AND PLANE

Plane is the simplest of surfaces. There are two methods for finding intersections of polyhedron and plane: **edge method** and **face method**.

Edge method is based on **intersection of line and plane** (Fig. 7.1).

Face method is based on **intersection of planes** (Fig. 7.1).



$$1 \rightarrow SA \times \Sigma$$

$$2 \rightarrow SC \times \Sigma$$

$$3 \rightarrow SB \times \Sigma$$

Edge method

Figure 7.1

$$12 \rightarrow SAC \times \Sigma$$

$$23 \rightarrow SBC \times \Sigma$$

$$13 \rightarrow SAB \times \Sigma$$

Face method

Face method is used if solid faces are particular position planes (Fig. 7.2). Figure 7.2 shows a prism, intersected by a plane Σ ; it shows also the development of the truncated prism. It is not necessary to do ancillary construction because there are true lengths of all edges on projections. Method of superposition is used for construction of true shape of intersection.

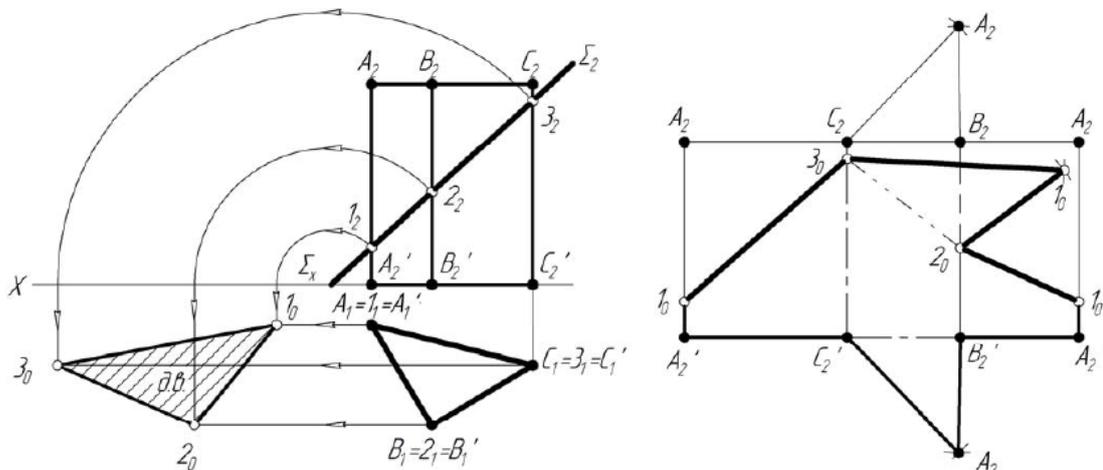


Figure 7.2

Edge method is illustrated on Figure 7.3. It shows a pyramid, intersected in a similar way by a plane Σ ; it shows also true shape of intersection and development of the truncated pyramid.

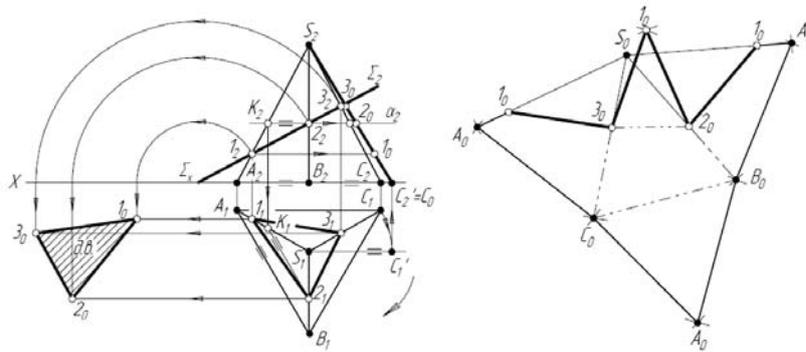


Figure 7.3

It is necessary to find the true length of the sides of the triangle SAC . The horizontal projection S_1C_1 does not show the true length of the line; neither does the frontal projection S_2C_2 . Let the pyramid be revolved about its axis, so that S_1C_1 will move to the line $S_1C'_1$ will be parallel to Π_1 ; and its projection on Π_2 , when found, will show the true length of the line. In revolving, the apex S will not move, and the point C will stay on C'_2 ; hence by, will be projected from C'_1 to the plane of the base, and will be the true length of the line SC .

All of the sloping edges of the pyramid are evidently of the same length, and the edges of the base are shown in plan in their true lengths; so the development may be constructed, laying down first the true size of the triangle SAB , joining to the edge SB the next face SBC , and then in turn the other face.

The triangle, representing the base, may be constructed on any side, as at BC . The most expeditious construction is to take a radius equal to strike an arc through A from center S , and on this arc, with the dividers or compasses, step off the length A_1C_1 three times, obtaining B , C , and A . These points are then connected and joined with S .

The lines on the development showing the intersection of the plane Σ are next found. The point I , for example, will appear in development on the line SA at its real distance from S . This real distance from S may be found in elevation by drawing a horizontal line from I_2 to cut in point thus determining $S_2C'_2$, as the real distance. The reason for this is that if the pyramid be revolved about its axis until S_1A_1 takes the position $S_1C'_1$, the frontal projection will then coincide with and the point I will appear on at the same height as before revolution. The length is then laid off from S , giving I . The real length from S to 2 is similarly found by a horizontal line from 2_2 to $2'_2$. The points I , 2 , 3 and I are then located and joined, thus completing the required development.

In connection with the construction for finding the true length of the line it should be remarked that it is not really necessary to think of the entire pyramid as revolving, for the same result is reached if the line alone be revolved about a vertical axis through S .

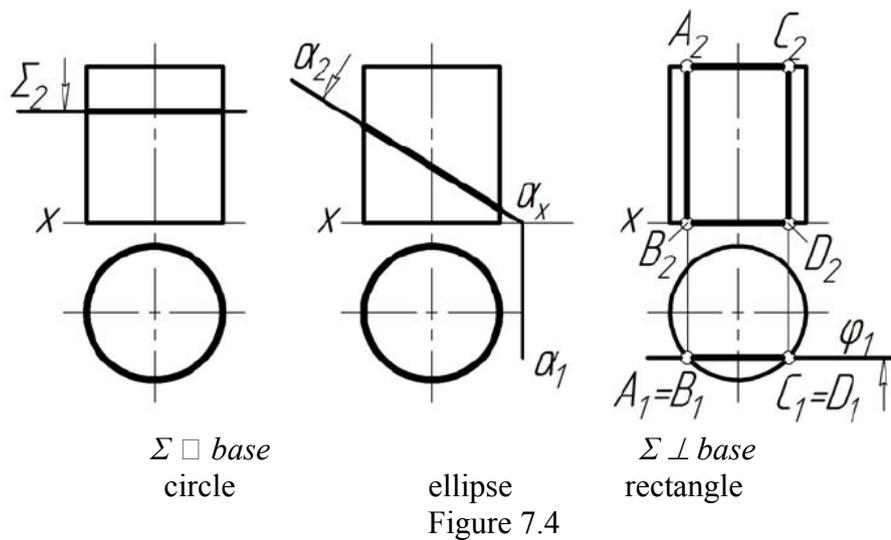
7.2 INTERSECTION OF A PLANE WITH A CURVED SURFACE

It has been shown that the intersection of a plane with a pyramid or prism is found by connecting points in which the plane cuts the edges of the solid. In the case of a solid with a curved surface, as a cone or cylinder, the only edges are the base edges. There may, however, be straight or curved lines drawn on the surface, and the intersection of the plane with these lines be found, thereby locating points on the required curved intersection.

7.2.1 INTERSECTION OF SURFACE OF REVOLUTION AND PLANE

Intersection of cylinder and plane

Kinds of intersection shape are presented in Figure 7.4.



A right circular cylinder standing on its base and cut by a plane Σ perpendicular to Π_2 , is drawn in Figure 7.5 a. The development and true shape of intersection of the cylinder are also shown. Elements of the cylinder are drawn, and the points noted in which the plane cuts the elements. The base is here divided into twelve equal parts, and twelve elements drawn. The plan of the intersection will coincide with the plan of the cylinder, since the whole convex surface is projected in the circle.

Development of the cylinder and of the curve of intersection.

Let the cylinder be placed with point of the base at corner point of the development, and element I at I_0 , on the plane of the paper (Fig. 7.5 b). Now imagine the cylinder rolled on the paper until the entire curved surface has come into contact with the paper, and element I is again on the paper at I_0 , parallel to I_0 . The distance between I_0 and I_0 is therefore the shortest distance around the cylinder as measured around the base.

As the cylinder rolls from I_0 to I_0 , the base remains always in a plane perpendicular to the elements, and hence perpendicular to the plane of the paper. Therefore, as the cylinder is developed, the edge of the base rolls out in the straight line drawn perpendicular to I_0 and extending to I_0 . The approximate distance around the base is laid off in the development, by taking the chord of one of the equal divisions of the base, and spacing it twelve times along base, locating the points. Lines drawn through these points at right angles to the developed base, will be the developed positions of the numbered elements. The lengths of the elements are taken directly from the elevation, since the true lengths are there shown; and a straight line through the upper ends of the elements will be the development of the top of the cylinder. The development of the curved surface of the cylinder is therefore the rectangle as shown.

The development of the section curve is obtained by locating the points $1_2, 2_2, 3_2$, etc., in the development on their respective elements, and at the true distance from the foot of the element. These distances are all shown in the elevation, and consequently may be transferred at once to the development; and a smooth curve drawn through $1_0, 2_0 \dots 5_0 \dots I_0$ will be the developed curve.

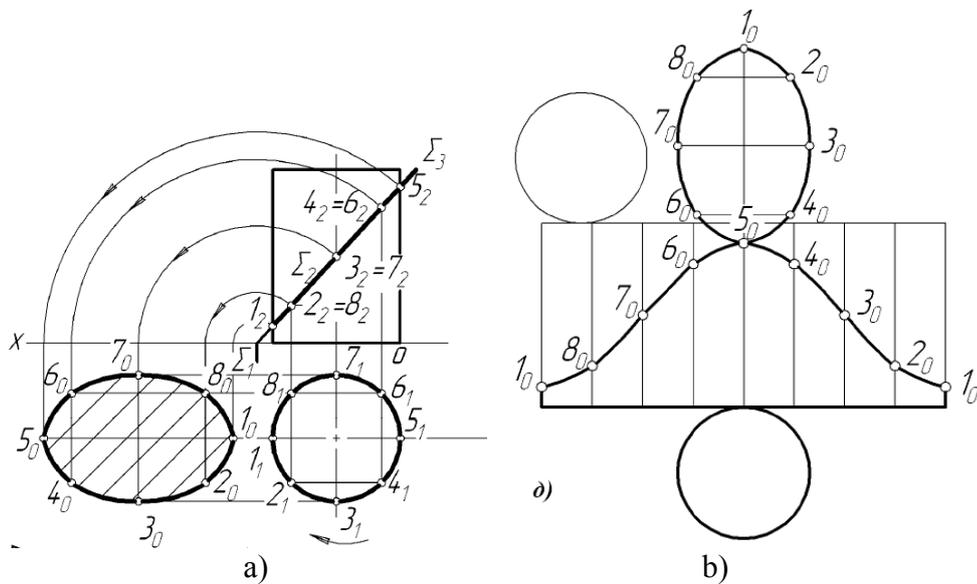


Figure 7.5

7.2.2 INTERSECTION OF CONE AND PLANE

The intersection of a cone of revolution and a plane is an **ellipse** if the plane (not passing through the vertex of the cone) intersects **all generators** (Fig. 7.6).

Let the plane of intersection α (α_2) be a frontal projecting plane that intersects all generators (Fig. 7.7). The endpoints of the major axis are A and B , the piercing points of the leftmost and rightmost generators respectively. The midpoint L of AB is the center of ellipse. Horizontal auxiliary plane β (β_2) passing through L intersects the cone in a circle with the center of K . (K is a point of the axis of the cone). The endpoints of the minor axis C and D can be found as the points of intersection of the circle in β and the reference line passing through L_2 .



Figure 7.6

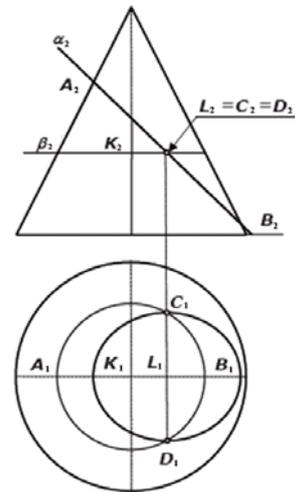


Figure 7.7

The intersection of a cone of revolution and a plane is a **parabola** if the plane (not passing through the vertex of the cone) is **parallel to one generator** (Fig. 7.8).

Let the plane of intersection α (α_2) frontal projecting plane be parallel to the rightmost generator (Fig. 7.9). The vertex of the parabola is V . Horizontal auxiliary plane β (β_2) can be used to find P_2 , the second image of a point of the parabola.

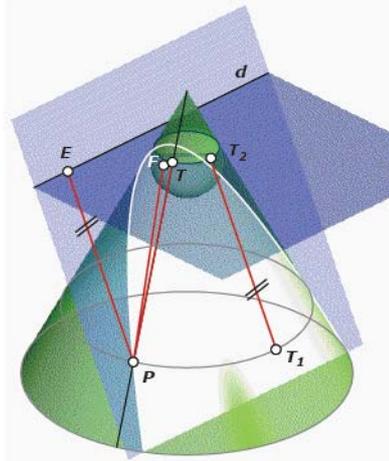


Figure 7.8

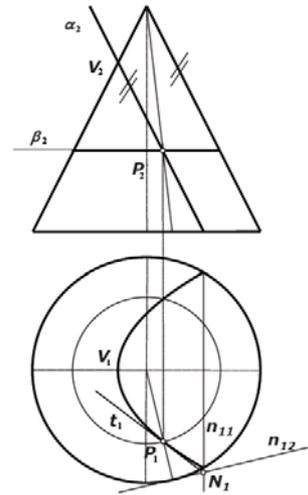


Figure 7.9

The intersection of a cone of revolution and a plane is a **hyperbola** if the plane (not passing through the vertex of the cone) is **parallel to two generators** (Fig. 7.10).

Let the plane of intersection α (α_2) frontal projecting plane parallel to two generators, that means, parallel to the frontal projecting plane β (β_2) through the vertex of the cone, which intersects the cone in two generators g_1 and g_2 (Fig. 7.11). The endpoints of the traverse (real) axis are A and B , the piercing points of the two extreme generators. The midpoint L of AB is the center of hyperbola.

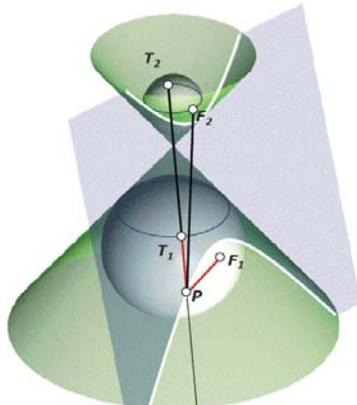


Figure 7.10

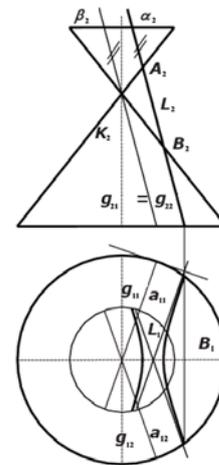


Figure 7.11

In Figure 7.12 is shown a right circular cone standing on its base, and intersected by a plane Σ , which is at right angles with Π_2 . The base may be divided into any number of parts (in this case eight) and the elements of the cone drawn, as $S-A$, $S-B$, $S-C$, etc. These elements are cut by the plane in points showing in elevation as 1 , 2 , 3 , etc., and these points are then projected to corresponding elements in plan.

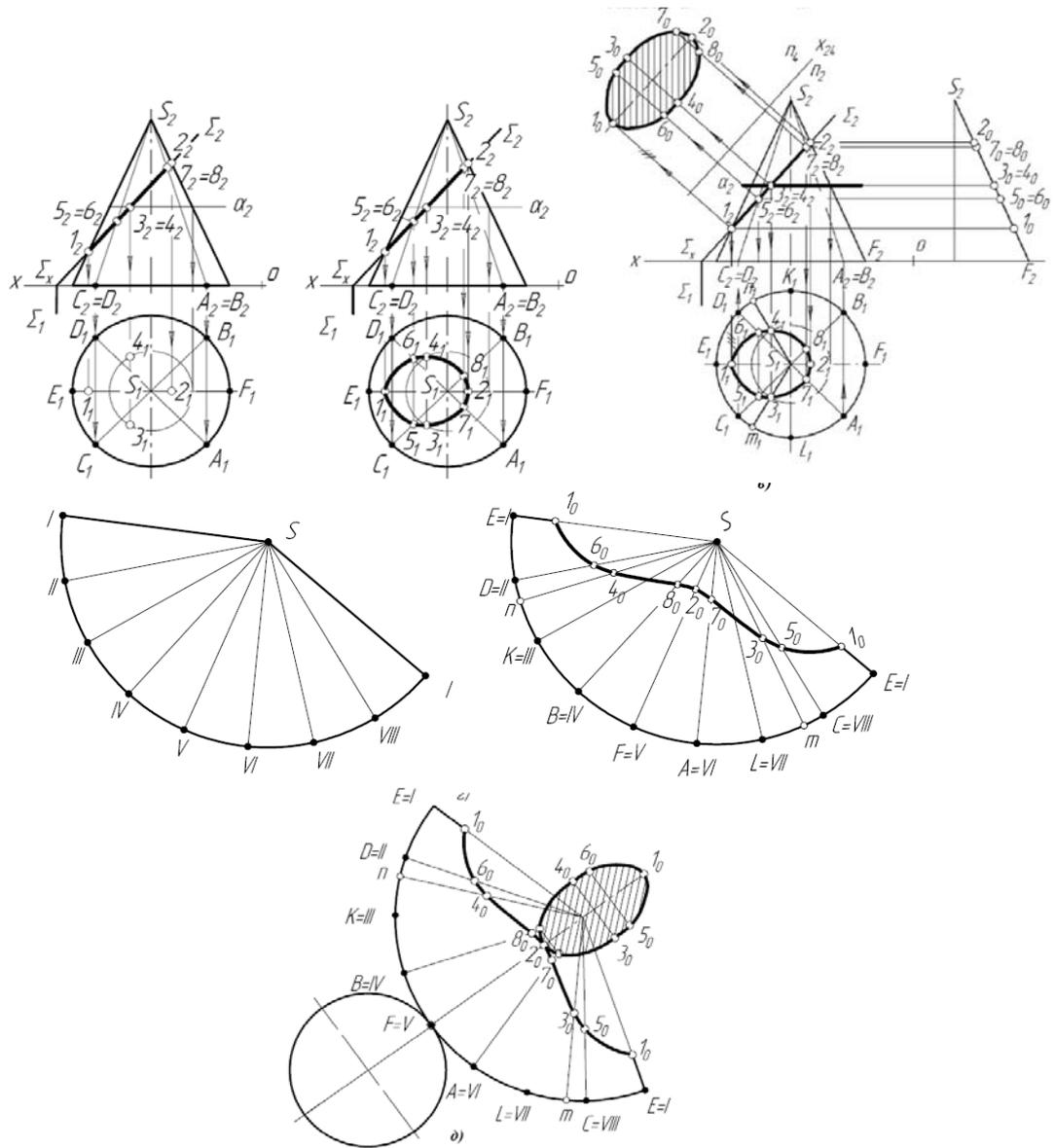


Figure 7.12

The points **3** and **4** are located in the plan as follows: Pass a plane α (α_2) through the points 3_2 and 4_2 , and parallel to the base of the cone. Then α will cut the cone in a circle whose diameter is shown on Fig. 68; and this circle, lying on the surface of the cone, will evidently pass through the **3** and **4**. The circle is drawn in plan, and, by its intersection with the projectors fixes the position of the points 3_1 and 4_1 . A smooth curve drawn in plan through the points **I**, **2**, **3**, **4**, etc. is the projection of the required intersection. The section is an ellipse, and its real size and shape are not in plan. Real size and shape of the ellipse are constructed by secondary projections.

We shall now proceed to the development of the cone and the section cut by plane Σ . The cone may be supposed to be placed with its vertex at **S** and the element **SE** at **S-I**; the cone is then rolled until whole of the curved surface has come into contact with the paper. As the cone rolls on the plane, the various elements will take positions such as **S-II**, **S-III**...**S-V** to **S-I**, and the edge of the base will develop into the curve passing through the points **I**, **II**, ... **I**. As all of the elements of the cone are the same length, the points **I**, **II**, **III**, etc., will all be at the same distance from **S**; that is, they will lie on the arc of a circle struck from **S** as center. The radius of the circle is the true length of any element of the cone, as shown by **SE** in elevation. The positions of the points **D**, **K**, etc., are found by taking in the dividers the distance between any two consecutive points of the base as seen on II_1 , and laying this off from **I** eight times. Lines joining these points with **S** will represent the positions of the elements in development.

As regards the development of the section cut by plane Σ , the different points of the curve may be found on the elements by laying off from S the true distance from the points of intersection to the vertex S . The distance S_2F_2 in elevation is a true length, since SF is parallel to Π_2 . The point 2 is then located at once. The true distance of the other points from the apex may be found by projecting horizontally from the points in elevation to S_2F_2 , on which line their true distances from S are shown. These distances are then laid off from S on the respective elements; and a smooth curve drawn through the points will be the section curve required.

Attention is called to the fact that the length of the developed base $I, II, III, \dots I$ etc. is not exact, since, in the first place, it is impossible with the dividers to measure exactly the length of a curve; and in the second place, because it is likewise impossible to apply exactly with the dividers any given distance along a curve. It should be said, however, that by taking points sufficiently close together a very good approximation indeed can be obtained.

Figure 7.13 shows an example of a cone standing on its base and intersected by a plane Σ perpendicular to Π_1 .

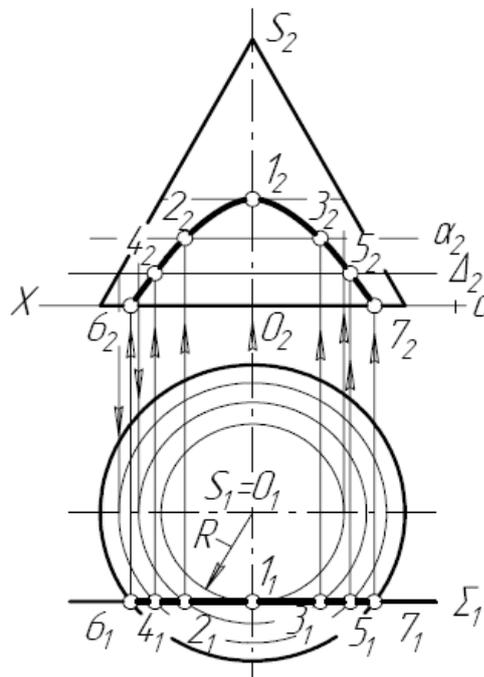


Figure 7.13

Elements of the cone are drawn as before, except that it is unnecessary to draw any on the further side of the cone. In plan, the elements are seen to intersect plane Σ in points $2, 3, 4, 5, 6$ and 7 , which are then projected to the elevation onto the corresponding elements. The highest point 1 cannot be found in elevation by direct projection, but is located by a special construction.

A circle with center S_1 is drawn on the plan passing through I_1 . Considering this circle as lying on the surface of the cone, it will intersect all of the elements, and will lie in a plane parallel to the base. For convenience, the points in which the circle intersects the two outside elements, are used, and projected to corresponding points in elevation; then the cut plane is the plane containing the circle, and I_2 in elevation is where this plane cuts corresponding generator. This section curve is shown in its true size and shape in elevation, and is known as a hyperbola, this name being given to the curve cut from any cone by a plane parallel to the axis.

7.3 THE INTERSECTION OF SURFACES

Intersection of surfaces is the set of all intersection points on one surface with the other surface.

Interpenetration of solids produce closed loops which may be made straight lines or curves.

Interpenetration of solids containing plane surfaces (prism with prism, pyramid with pyramid, prism with pyramid) results in a polygonal line or polygonal lines.

The type of intersection of surfaces depends on the types of geometric forms, which can be two- or three-dimensional.

Solids having curved surfaces results in closed curve or curves.

The type of intersection depends on the types of geometric forms and their geometric relationship. There are several geometric relationships:

external-contact of surfaces– Common element is point or line surface contact if one line on one surface intersect the one line on other surface (Fig. 7.14);

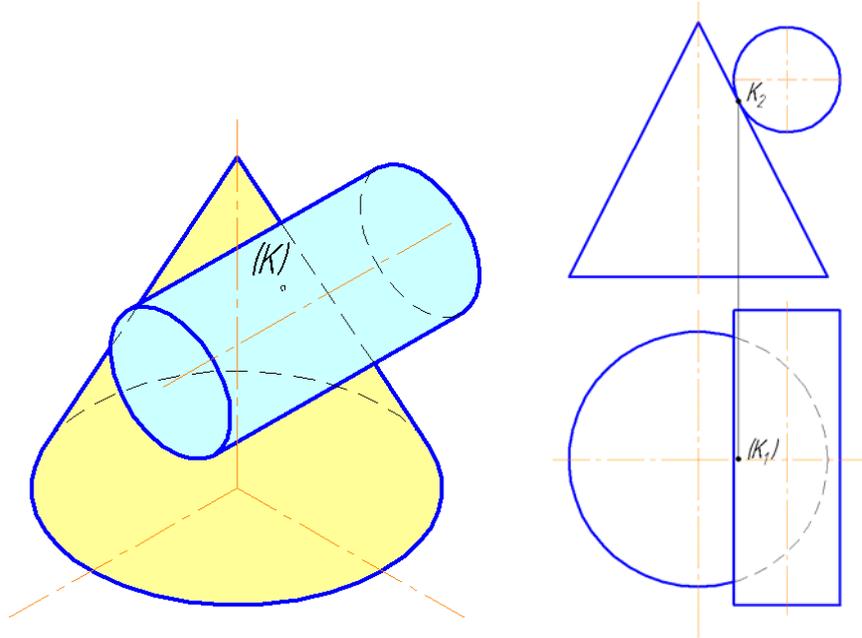


Figure 7.14

internal-contact of surfaces–Line of intersection is criss-cross closed line (Fig. 7.15);

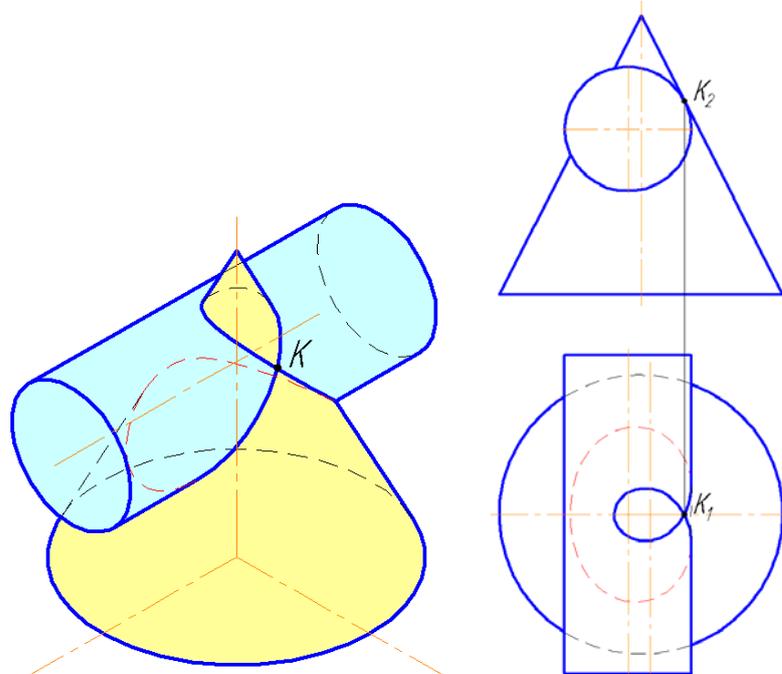


Figure 7.15

full penetration (total intersection) of one surface into other - Intersection is two closed lines if all lines on one surface intersect the other surface (Fig. 7.16, 7.17);

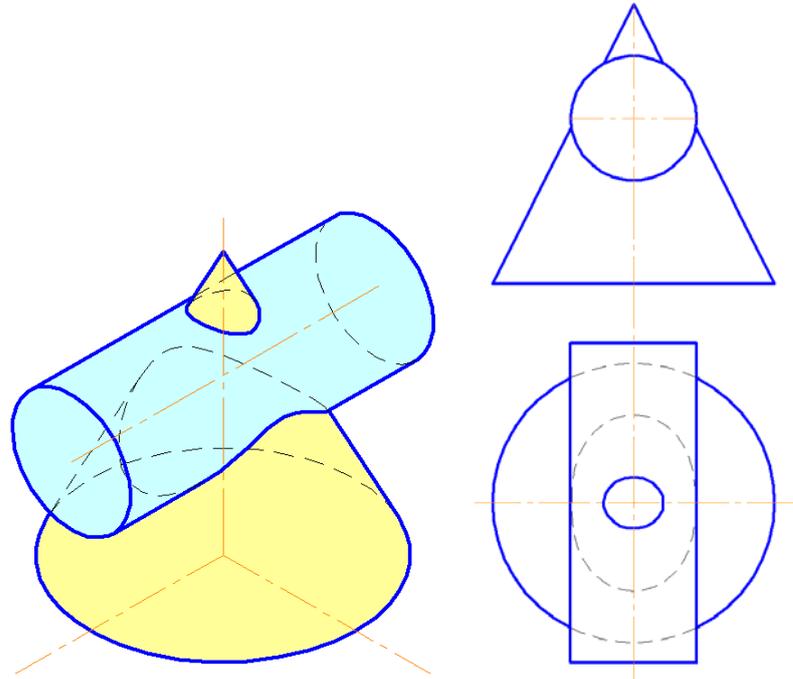


Figure 7.16

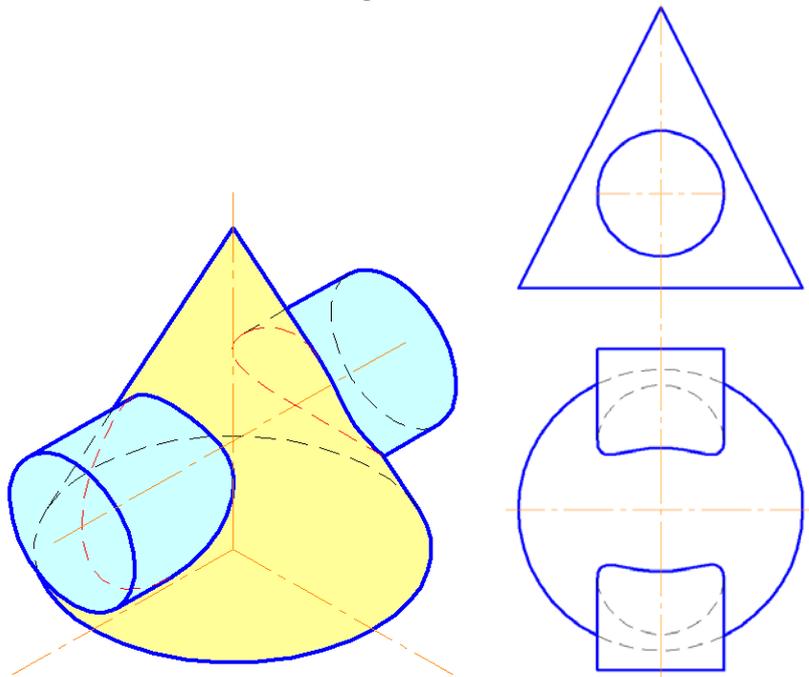


Figure 7.17

partial penetration (partial intersection) of surfaces - Line of intersection is one closed line (Fig. 7.18). In the case of a partial intersection, on one of the surfaces there are lines not intersecting the other surface.

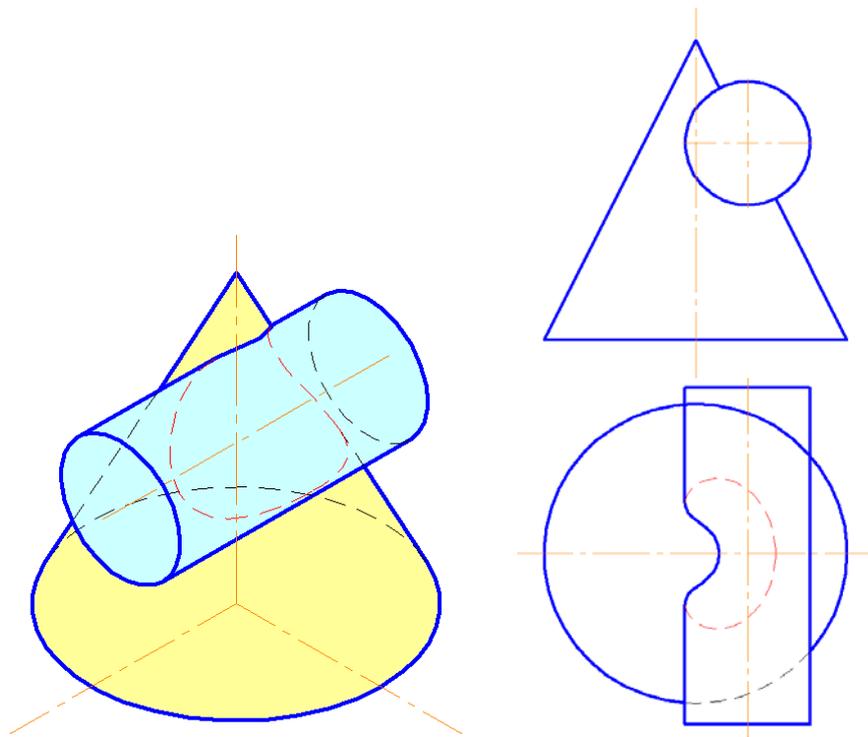


Figure 7.18

The line of intersection of two surfaces is found by determining a number of points common to both surfaces and drawing a line or lines through these points in correct order. This direction is the same for all projections bodies. All **critical points** of intersection must be founded. The resulting line of intersection may be straight, curved, or straight and curved. The problem of finding such a line may be solved by one of two general methods, depending on the type of surfaces involved. For the purpose of simplifying this discussion of intersections, it should be assumed that all problems are divided into these two general groups:

- Problems involving two geometric figures, both of which are composed of plane surfaces.
- Problems involving geometric figures which have either single-curved or double-curved surfaces.

Any practical problem resolves itself into some combination of the geometrical type forms.

7.4 INTERSECTION OF SOLIDS BOUNDED BY PLANE SURFACES (POLYHEDRONS)

For instance, the procedure for finding the line of intersection of two prisms is the same as that for finding the line of intersection of a prism and a pyramid; hence, both problems belong in the same group.

Interpenetration of polyhedrons is polygonal closed line or lines.

Problems of the group are solved by locating the points through which the edges of each of two geometric shapes pierce the other. These points are **vertices** of the line of intersection (Fig. 7.19).

There is intersection of two prisms in Figure 7.20. Whenever one of two intersecting plane surfaces appears as a line in one view, the points through which the lines of the other surfaces penetrate it usually may be found by inspecting that view.

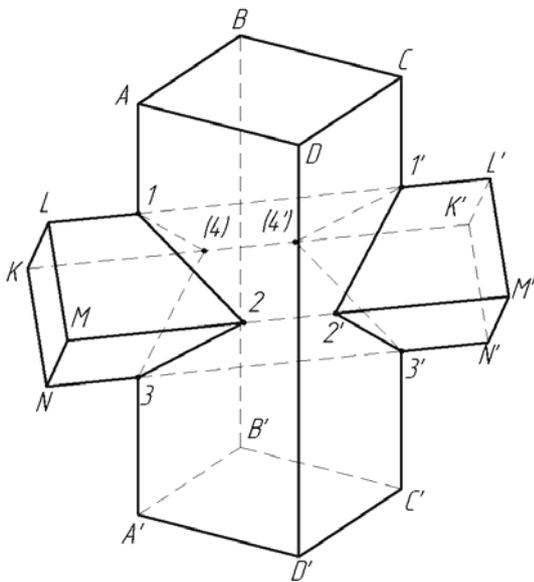


Figure 7.19

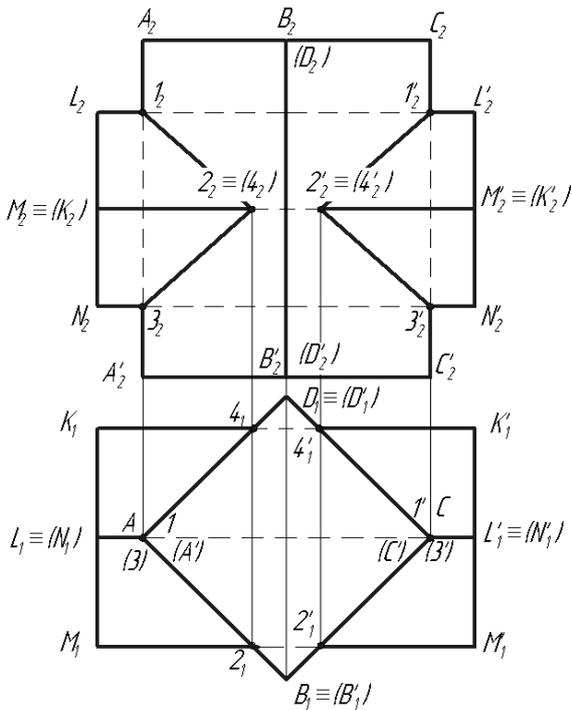


Figure 7.20

The faces of the vertical prism are seen as lines in the top view. Lines LL' and NN' intersect the edge of the vertical prism at points 1 and 3 (coinciding with AA'). Lines MM' and KK' intersect the faces at 2 and 4 respectively.

The exact positions of these points along the length of the prism may now be determined by projecting them on corresponding lines in the front view. For example, 2_1 is projected to 2_2 on the line $M_2M'_2$. Note that 4_2 coincides with 2_2 .

7.5 INTERSECTION OF CURVE SURFACES

Since the problem of finding the line of intersection of two cylinders and the problem of finding the line of intersection of a cylinder and a cone both involve single-curved surfaces, these two also belong in the same group.

Problems of this group may be solved by drawing elements on the lateral surface of one geometric shape in the region of the line of intersection. The points at which these elements intersect the surface of the other geometric shape are points that are common to both surfaces and consequently lie on their line of intersection. Intersections must be represented on multi-view drawings correctly and clearly. For example, when a conical and a cylindrical shape intersect, the type of intersection depends on their sizes and on the angle of intersection relative to their axes.

A curve, traced through these points with the aid of a French curve, will be a representation of the required intersection. To obtain accurate results, some of the elements must be drawn through certain **critical** points at which the curve changes sharply in direction. These points usually are located on contour elements. Hence, the usual practice is to space the elements equally around the surface, starting with a contour element.

The intersection of two solids, viewed from any given direction, is in general partly visible and partly invisible. The rule for visibility is that visible parts of the intersection must always be the intersection of visible parts of each surface. Thus the intersection of the upper parts of two surfaces would be visible in plan; while the intersection of the upper side of one with the under side of the other would not be visible.

The line of intersection is determined using auxiliary views and auxiliary surface method.

7.5.1 AUXILIARY SURFACE METHOD

System of auxiliary surfaces can be chosen as system of elementary surfaces. Auxiliary surfaces can be **planes** or **spheres**.

There are **cutting-plane method** and **cutting-sphere method (concentric cutting- sphere method and eccentric cutting- sphere method)**.

Method of construction of the intersection curve depends on the type of intersecting surfaces.

System of auxiliary surfaces can be chosen in such way, that the intersection curves of surfaces and both given surfaces are elementary curves (lines, or circles).

Arbitrary point **1** (and **2**) on the intersection curve is the point located on both surfaces Σ and Δ , it is therefore located on some curve k on the surface Σ and on some curve l on the surface Δ , while $k \cap l = I$ (Fig. 7.21).

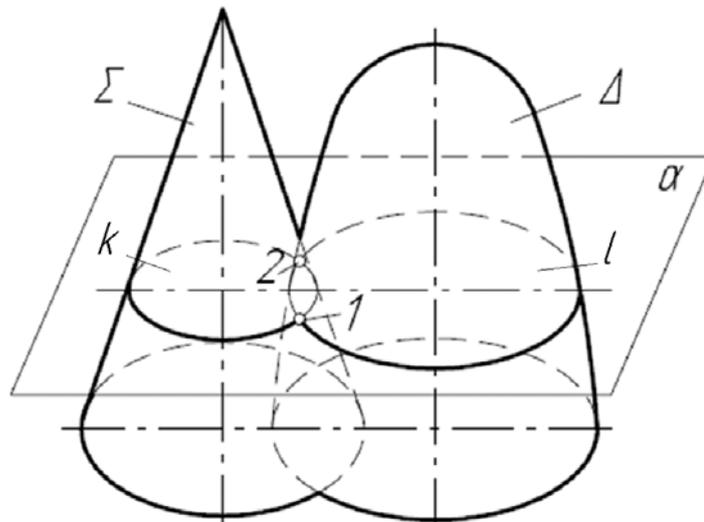


Figure 7.21

Curves k, l are located on one surface α , which intersects surface Σ in the curve k and surface Δ in the curve l .

In the construction of points on the intersection curve of surfaces Σ and Δ we choose the system of auxiliary surfaces, for which the following is valid:

Common points of curves k and l (if they exist) are points on the intersection curve.

Auxiliary surfaces α from the system are usually aptly chosen planes or spheres, with regard to the intersecting surfaces Σ and Δ , in order to receive points on the intersection in the most precise and easy way.

Cutting-plane method: The two solids are assumed to be cut by a series of cutting planes. The cutting planes may be vertical (i.e. perpendicular to the Π_1), edgewise (i.e. perpendicular to the Π_2) or oblique. The cutting planes are so selected as to cut the surface of one of the solids in straight lines and that of the other in straight lines or circles.

In general the method of finding the line of intersection of any two surfaces is to pass a series of planes through them in such a way as to cut from each the simplest lines. The intersection of the lines cut from each surface by a plane will give one or more points on the line of intersection.

The intersection of cone and sphere is shown in Fig. 7.22.

Auxiliary cutting plane passes at right angle to the axis of the cone and parallel to its base.

Points **1** and **2** are at the intersection of outlines of cone and sphere. Points **1** and **2** are uppermost and lowermost points of the curves of intersection. Points **5** and **6** are constructed by plane Σ . Points **3** and **4** are constructed by plane α . Points **3** and **4** are points at which curve changes visibility on horizontal projection.

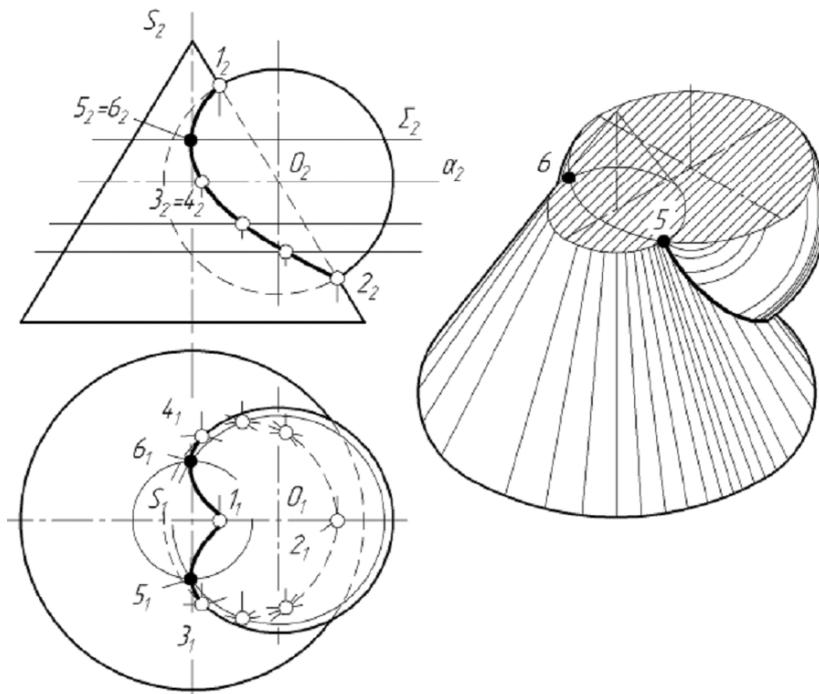


Figure 7.22

7.5.2 INTERSECTIONS OF SPHERE AND ELEMENTARY SURFACES

Before beginning to consider cutting-sphere method it is well also to pay attention to intersections of sphere and elementary surfaces.

If centre O of the sphere Φ is not located on the axis of the surface of revolution, intersection must be constructed as intersection of two surfaces of revolution (Fig. 7.22 and Fig. 7.23).

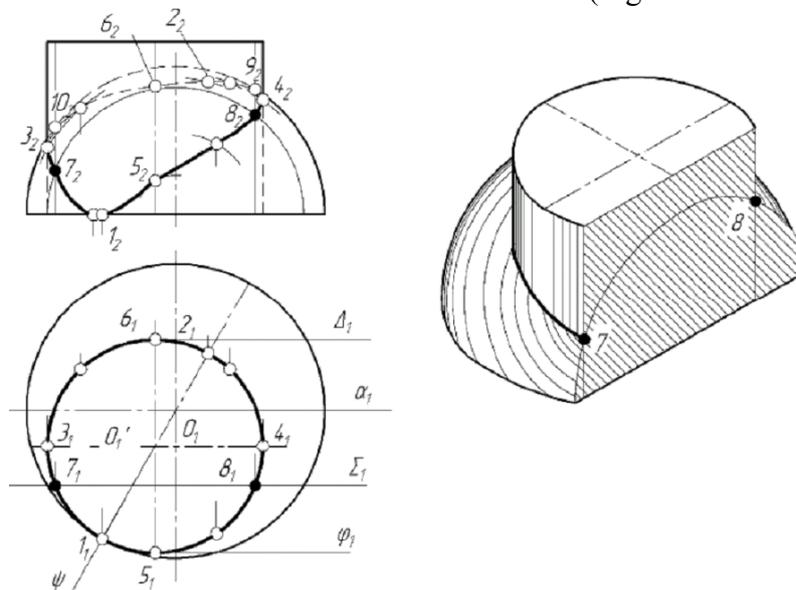


Figure 7.23

Intersection of sphere and pyramidal or prismatic surface is a curve composed from circular arcs, which are intersections of sphere by separate faces of these surfaces (Fig. 7.24).

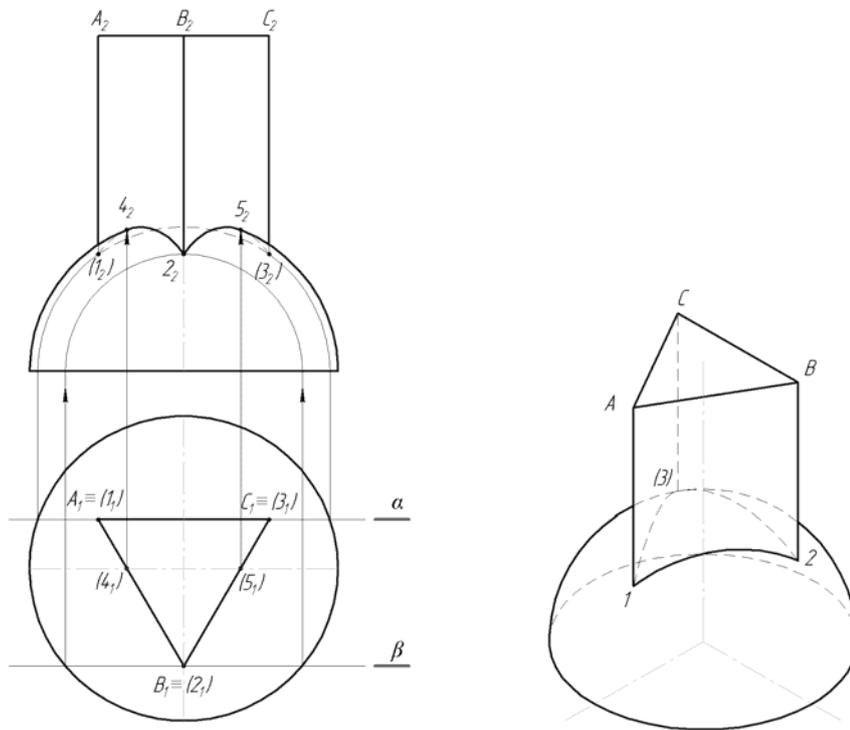


Figure 7.24

Intersection of two spheres $\Phi = (S, r)$, $\Psi = (S', R)$ is

empty set, if $|SS'| > r + R$;

one single point, if $|SS'| = r + R$;

circle k , if $|SS'| < r + R$.

Circle k is located in the plane perpendicular to the line SS' and its centre is on this line.

Orthographic view of k to the plane parallel to the line SS' is line segment perpendicular to the view of line SS' (Fig. 7.25).

One point of the intersection circle k is, for example, the common point M on outlines of spheres $\Phi = (S, r)$, $\Psi = (S', R)$.

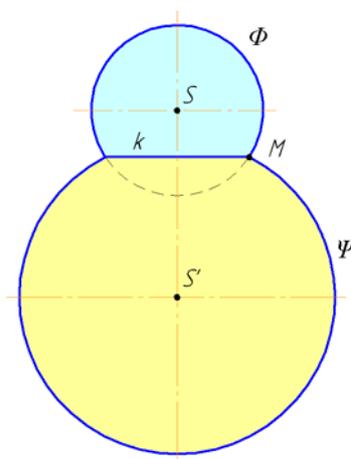


Figure 7.25

If centre of the sphere $\Phi = (S, r)$ is located on axis of the conical or cylindrical surface of revolution, intersection is a pair of circles located in planes perpendicular to the axis of the surface of revolution.

These are mapped orthographically to the plane parallel to the axis as line segments (Fig. 7.26).

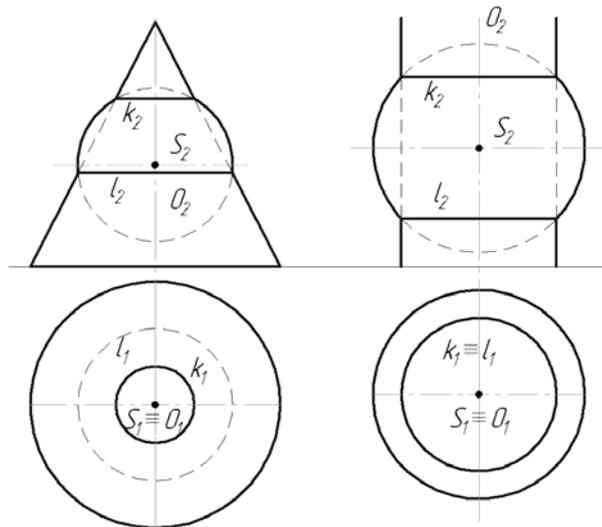


Figure 7.26

Intersection of sphere and conical or cylindrical surface, which is not the surface of revolution, can be determined as a set of intersection points of lines on the concerned surface with the sphere by means of the system of auxiliary planes.

7.5.3 CUTTING-SPHERE METHOD

Cutting-sphere method divides into concentric cutting-sphere method and eccentric cutting-sphere method.

Concentric cutting- sphere method

Intersections of surfaces of revolution with intersecting axes.

Intersection point O of axes of two surfaces of revolution is the centre of the spheres in the system (Fig. 7.27).

Type of intersection of surfaces can be determined by the tangent sphere.

Full penetration (total intersection) of cone into cylinder is if the least tangent sphere is the tangential to cone (Fig. 7.27, a).

Full penetration (total intersection) of cylinder into cone is if the least tangent sphere is the tangential to cylinder (Fig. 7.27, c).

Internal-contact of surfaces is if tangent sphere is the tangential to cone and cylinder (Fig. 7.27, b, in this case – **double internal-contact**).

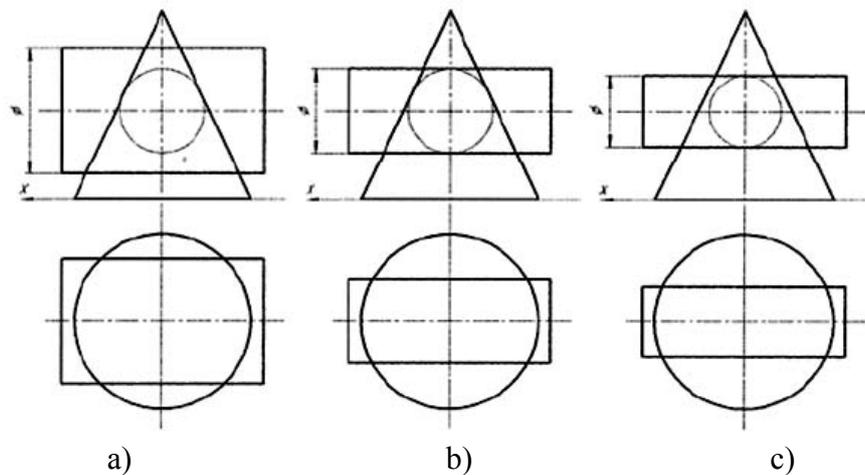
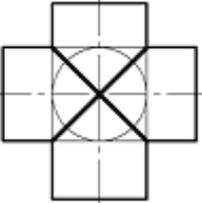
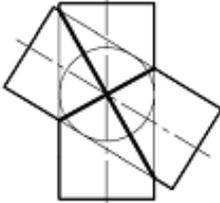
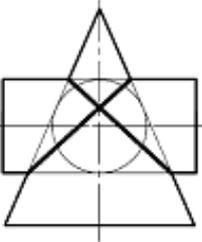
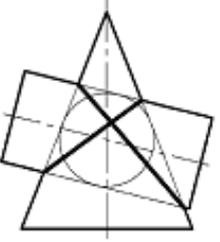
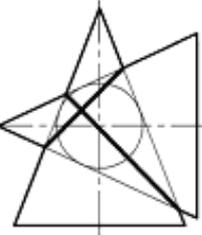
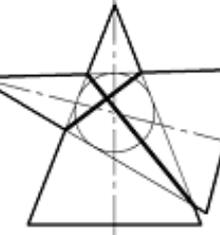


Figure 7.27

Double internal-contact of surfaces is special case. In this case intersection is two closed plane lines (ellipses) with common points. There are some intersections of surfaces in Tabl. 1.

Tabl. 1

Cylinder and cylinder		
Cylinder and cone		
Cone and cone		

Any of the spheres intersects both surfaces in parallel circles. Common points of these parallel circles are points on the intersection curve (Fig. 7.28).

Centre of the inscribed sphere O is in the intersection point of axes of intersecting surfaces of revolution.

One of the spheres g shares with intersecting surfaces (cone and cylinder) common circles, projected on frontal projections as line segments: with cylinder - $1_22_2, 3_24_2$; with cone - $5_26_2, 7_28_2$.

Their common points K, K', M, M' are located on the intersection

$$1_22_2 \cap 5_26_2 = K_2, 1_22_2 \cap 7_28_2 = K'_2, 3_24_2 \cap 5_26_2 = M_2, 3_24_2 \cap 7_28_2 = M'_2.$$

Other points on the intersection can be found as the intersection points of outlines: A_2, B_2, C_2, D_2 .

Sphere g_{min} is tangent to one of intersecting surfaces and intersecting one to other surface.

Horizontal projections of the points K_1, K'_1, M_1, M'_1 can be determined as points of the horizontal projection of circle 5_16_1 .

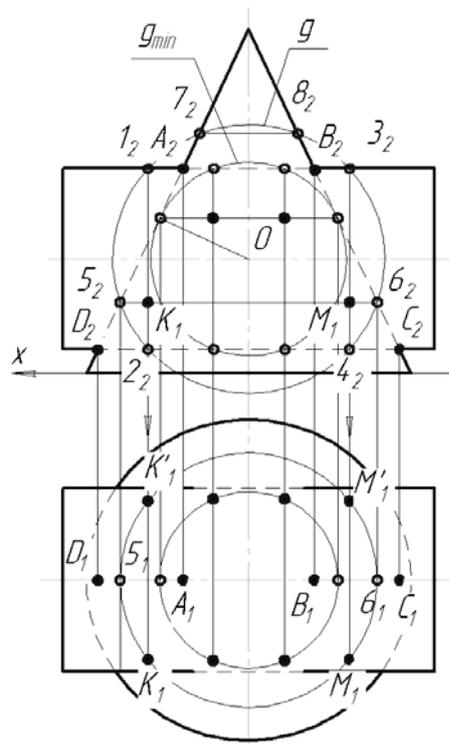


Figure 7.28

In the Fig. 7.29 there is the intersection of two cones of revolution with intersecting axes.

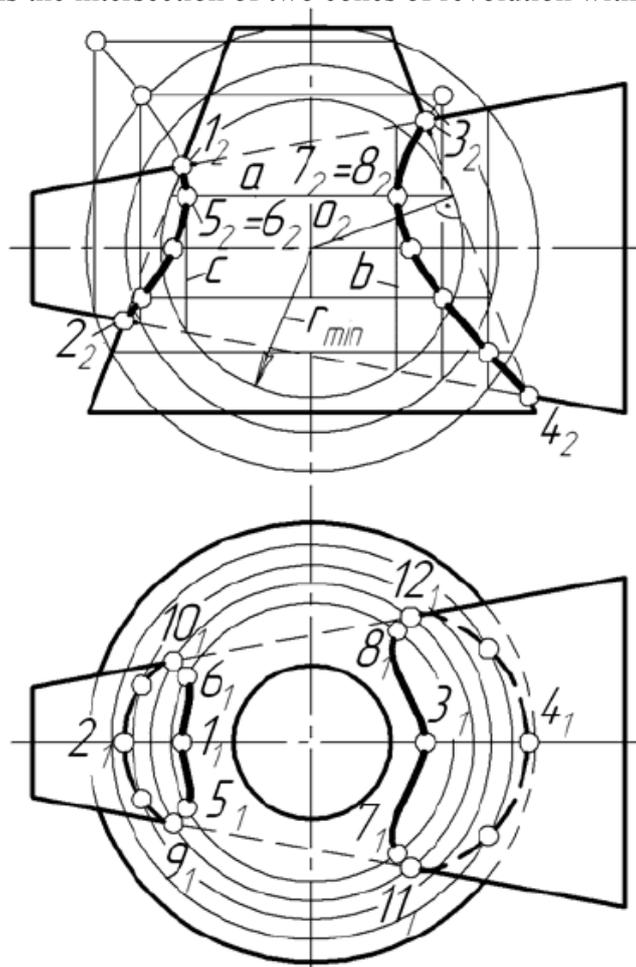


Figure 7.29

Eccentric cutting- sphere method

Intersections of surfaces of revolution with skew axes.

In the construction of the intersection of torus and conical surface of revolution, the system of spheres with the centers on the axis of the conical surface is used (spheres intersect conical surface in circles) (Fig. 7.30).

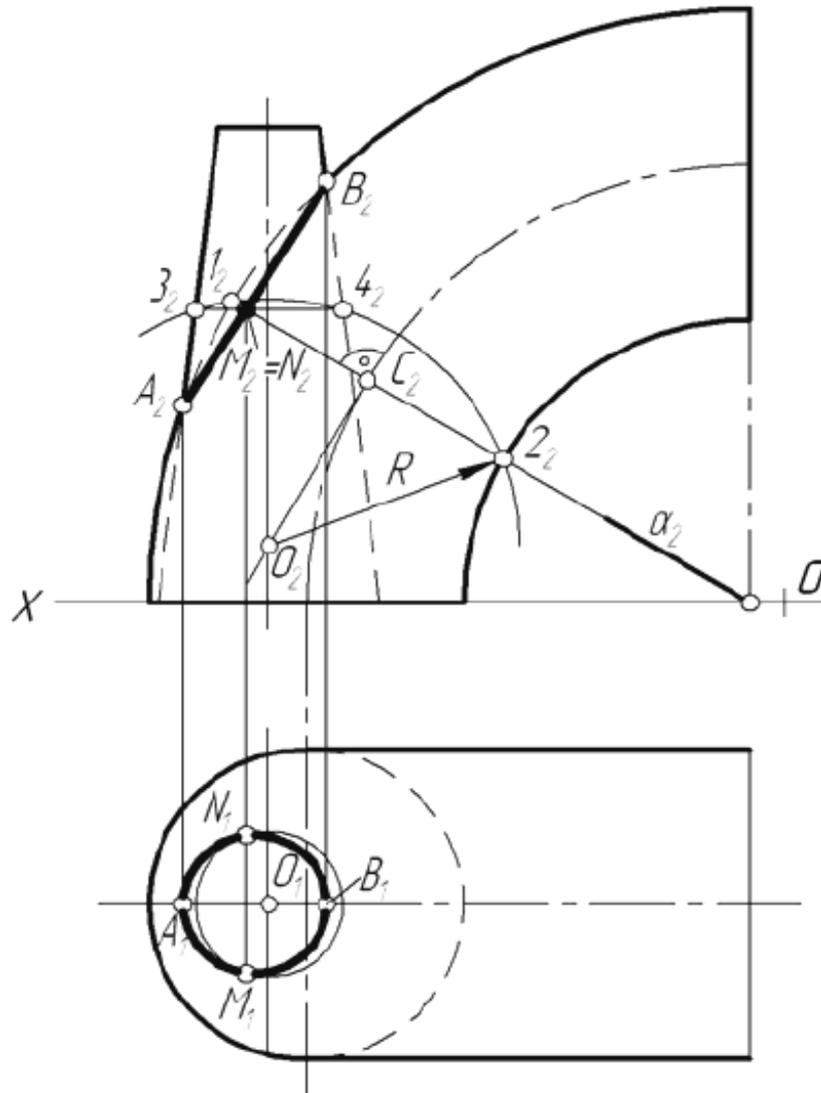


Figure 7.30

Choosing a circle $12(I_2 2_2)$ on the torus (Fig. 7.30), which is an intersection by the meridian plane $\alpha(\alpha_2)$ projected to the image plane as a line segment, the sphere from the system is determined.

Line passing through the circle $12(I_2 2_2)$ centre C_2 (on the axial torus circle) and perpendicular to the meridian plane $\alpha(\alpha_2)$ intersects the axis of conical surface in the centre $O(O_2)$ of the sphere ($12(I_2 2_2)$ is its planar intersection by $\alpha(\alpha_2)$).

Sphere intersects conical surface in parallel circles (one of them is $34(3_2 4_2)$) projected to line segments.

Common points of circles $12(I_2 2_2)$ and $34(3_2 4_2)$ are $M(M_2)$ and $N(N_2)$ on the intersection curve.

Points $A(A_2)$ and $B(B_2)$ are points of intersection of surfaces outlines.

Horizontal projections of all points of intersection are constructed as the points of conical surface of revolution.

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УДК 514.18
ББК 30.11
Т 19

Рецензент:

Шкіца Л. Є. докт. техн. наук, завідувач кафедри інженерної та комп'ютерної графіки

*Рекомендовано методичною радою університету
(протокол № _____ від _____ 2013р.)*

Тарас І. П.

Т 19 Нарисна геометрія, інженерна та комп'ютерна графіка. Частина 1. Нарисна геометрія: конспект лекцій – Івано-Франківськ: ІФНТУНГ, 2013. - 81 с.

МВ 02070855- - 2013

Конспект лекцій складено відповідно до робочої програми з дисципліни «Нарисна геометрія, інженерна та комп'ютерна графіка». Призначений для студентів напряму підготовки – 6.050304 - Нафтогазова справа.

Містить теоретичні відомості про методи проєціювання, проектування елементарних геометричних фігур, методи рішення позиційних та метричних задач, способи перетворення проєкцій, поверхні та їх взаємний перетин.

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та комп'ютерної графіки

Л. Є. Шкіца

Узгоджено: зав. кафедри розробки
та експлуатації нафтових і газових родовищ

Р. М. Кондрат

Голова експертно-рецензійної
комісії університету

І. В. Костриба

Нормоконтролер

Г. Я. Онуфрик

Інженер I категорії

Л. З. Костюк

УДК 514.18
ББК 30.11
МВ 02070855- - 2013

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