The Analysis of the Methods of Data Diagnostic in a Residue Number System

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Abstract. The article presents the results of the analysis of the methods of data diagnostic presented in residue number system (RNS). Two practical methods of data diagnostic in RNS are investigated. Their advantages and disadvantages are shown. The main disadvantage of these methods is the lack of the efficiency in data diagnostic in RNS. The third method of the efficient diagnostic in RNS, which eliminates the above-mentioned disadvantage, has been reviewed in the article. The usage of this method can significantly increase the efficiency of data diagnostic in RNS. The main drawback of this method is a significant amount of equipment required to implement the process of data diagnostic in RNS. The method of the efficient diagnostic has been improved in terms of reducing the amount of equipment required for implementing the process of data diagnostic in RNS. The application of the improved method of the efficient diagnostics allows reducing the amount of equipment for the implementation of a diagnostic data procedure in RNS without increasing the diagnostic time. Examples of practical use of the improved method of data diagnostic in RNS are presented. The results of the analysis of existing methods for diagnosing data presented in the RNS carried out in the article have scientific, theoretical and practical significance. Firstly, the results of the analysis make it possible to evaluate the theoretical state of the solution to the problem of data diagnostics in machine arithmetic in residual classes. Secondly, the examples of the implementation of diagnostic methods presented in the article, for a different set of grounds, confirm the practical applicability of the methods considered.
The significance and value of the materials of the presented article lies in the further development of the theory and practice of machine arithmetic in a non-positional number system in residual classes. The quality of the article corresponds to the academic level in the theory and practice of creating machine arithmetic in residual classes.

Keywords: Alternative Set of Numbers; Data Diagnostic; Diagnostic Efficiency; Error Control and Correction; Residue Number System; Zeroisation Procedure.

1 Introduction

Data diagnostic in residue number system (RNS) is the process of determining the distorted residues in redundant non-positional code structure (NCS) presented in the following form $A_{RNS} = (a_1 || a_2 || \dots || a_{i-1} || a_i || a_{i+1} || \dots || a_n || \dots || a_{n+k})$ where n and k are the number of, respectively, informational and control bases m_i $(i = \overline{1, n+k})$ of ordered $(m_i < m_{i+1})$ RNS. The diagnostic is carried out after data control, if it is necessary for the subsequent error correction. Some methods, algorithms and devices for data diagnostic in RNS have already been presented [1-3]. To monitor, diagnose and correct errors, the certain information redundancy must be introduced. Power R of the information redundancy, which as in positional number system (PNS), determines the corrective abilities of the code, is estimated by the value $d_{min}^{(RNS)}$ of a minimum code distance (MCD). In RNS the value of MCD is determined by the ratio $d_{min}^{(RNS)} = k + 1$ [4-7]. For one control base, the value of MCD is equal to $d_{min}^{(RNS)} = 2$. In accordance with the general coding theory, in RNS with a minimum code distance $d_{min}^{(RNS)} = 2$ the distortion of only one of the residues can be reliably established (one-time error) in NCS. For example, to correct a one-time error (in one residue) and determine double errors (in two residues) it is necessary to ensure that $d_{min}^{(RNS)} = 3$ [1, 8-12]. Due to the influence of RNS properties on the data processing it is possible, in some cases, to correct one-time data errors (in one NCS residue) when introducing the minimal (k = 1) information code redundancy. So, the property of the independence of the residues of NCS allows us to correct not intermediate calculation results, but final one. A typical example for this case is the possibility of implementing the data error correction procedure with one control base without stopping the intermediate computing process (during the computational process). To implement such procedure, it becomes necessary to diagnose intermediate results of calculations based on the use of the concept of an alternative set of numbers (AS) in RNS [13-19].

The purpose of the article is to study the methods of data diagnostic, presented in non-positional residue number system with one control base.

The methodology of the research conducted in the article is based on the use of the principles and methods of data processing presented in the residual classes.

Literature review. The results of research and literature review in the field of the the methods of data diagnostic of well-known authors (Valakh M., Svoboda A., Sabo N., Aksushskyi I.Y., Yuditskyi D.I., Glushkov V.M., Torgashov V.A., Amberbaev V.M., Kolyada A.A., Shimbo A., Paulier P., Thornton M.A., Dreschler R., Miller D.M., and others) showed that the use of RCS as a system of calculations of computer systems (CS), intended for the implementation of integer arithmetic operations of addition, subtraction and multiplication numbers in the positive numerical range, significantly increases the speed of the solution of problems of a certain class. In recent years, the following CSs have been developed in the RCS: On-board computer Star (USA); specialized DFT processors (USA, South Korea); a number of military specialized on-board computers (USA, Japan); specialized DSP processors (USA); Sprint Computers for Robotics (USA, Japan); in the Chinese company "Tpv Display Technology (Wuhan, China) Co., Ltd" in the development and implementation of a wireless sensor network monitoring system for industrial equipment in the manufacture of monitors; at the enterprise "Relcom-Podillya Ltd." in developing the system of video surveillance on the basis of wireless multimedia sensor networks; at "Cypress Semiconductors Corporation" in developing hardware software for CY8CKIT-050 PsoC 5 and CyFi (CYRF7936) modules that can be used in wireless sensor networks.

Discussion. Let us consider the method of data diagnostic in RNS based on the concept of AS numbers in RNS.

The first method of diagnosis. The alternative set $W(\tilde{A}) = \{m_{l_1}, m_{l_2}, \ldots, m_{l_p}\}$ of incorrect number $\tilde{A}_{RNS} = (a_1||a_2||\ldots||a_{i-1}||\tilde{a}_i||a_{i+1}||\ldots||a_n||a_{n+1})$ can be determined by a sequential testing of each base m_i $(i = \overline{1,\ n})$ RNS. We determine the set of numbers, that have the same residues for all bases of RNS, as number $\tilde{A}$, except one certain residue (base), and differ only in values of possible residues on this base. In this set there may be no correct numbers or there may be only one correct number. In the last case, the number is a part of AS of number $\tilde{A}$.

The proposed method involves carrying out similar verifications for each of the information base of RNS (a control base always is a part of a set of bases of AS). The result of such sequential verifications completely and reliably determines the AS $W(\tilde{A}) = \{m_{l_1}, m_{l_2}, \ldots, m_{l_\rho}\}$ of the incorrect number $\tilde{A}$. The disadvantage of the method is the low efficiency in determining AS. This is due to the considerable time of consecutive executions of data diagnostic stages in RNS.

The second method of diagnosis. This method is also based on the determination of AS $W(\tilde{A}) = \{m_{l_1}, m_{l_2}, \ldots, m_{l_\rho}\}$. In this case, the whole procedure of diagnosing NCS is carried out by simultaneous and parallel calculation of all possible projections $\tilde{A}_{i\ RNS} = (a_1||a_2||\ldots||a_{i-1}||a_{i+1}||\ldots||a_n||a_{n+1})$ of the incorrect number $\tilde{A}_{RNS} = (a_1||a_2||\ldots||a_{i-1}||\tilde{a}_i||a_{i+1}||\ldots||a_n||a_{n+1})$, and their subsequent comparison with the value of $M = \prod_{i=1}^{n} m_i$ without the redundant numeric information interval (information volume of code words) $0 \div M - 1$ given in RNS. It is proved in [1, 7, 8], that the necessary and sufficient condition of the entry of the bases of RNS in AS $W(\tilde{A}) = \{m_{l_1}, m_{l_2}, \ldots, m_{l_\rho}\}$ of number $\tilde{A}_{RNS} = (a_1||a_2||\ldots||a_{i-1}||\tilde{a}_i||a_{i+1}||\ldots||a_n||a_{n+1})$ is correctness of $(\tilde{A}_{i\ RNS} < M)$ for its projection $\tilde{A}_{i\ RNS} = (a_1||a_2||\ldots||a_{i-1}||a_{i+1}||\ldots||a_n||a_{n+1})$. Parallelization of the procedure of calculating all possible projections $\tilde{A}_{i\ RNS} = (a_1||a_2||\ldots||a_{i-1}||a_{i+1}||\ldots||a_n||a_{n+1})$ of the incorrect number $\tilde{A}_{RNS} = (a_1||a_2||\ldots||a_{i-1}||a_i||a_{i+1}||\ldots||a_n||a_{n+1})$ reduces the time of AS determination and increases the efficiency of diagnosing data in RNS.

Let us consider the following example of data diagnostic based on the usage of the second method.

Example 1. Let us determine the AS of the number $\tilde{A}_{RNS} = (0||0||0||0||5)$, which is defined in RNS by the information $m_1 = 3$, $m_2 = 4$, $m_3 = 5$, $m_4 = 7$ and control bases $m_k = m_5 = 11$. Wherein $M = \prod_{i=1}^{n} m_i = \prod_{i=1}^{4} m_i = 420$ and the full range $0 \div M_0 - 1$ of coded words equals to $M_0 = M \cdot m_{n+1} = 420 \cdot 11 = 4620$ (Table 1).

Table 1. Table of code words

A in PNS	A in RNS					A in PNS	A in RNS				
0	0	0	0	0	0	2310					
1	1	1	1	1	1	2311					
2	2	2	2	2	2	2312					
3	0	3	3	3	3	2313					
. . .						. . .					
418						2728					
419						2729					
420						2730					
						. . .					
			. . .			***3360***	***0***	***0***	***0***	***0***	***5***
						. . .					
2308						4618					
2309						4619					

At first, the procedure of controlling number $A_{RNS} = (0 \parallel 0 \parallel 0 \parallel \; 0 \parallel \; 5)$ is carried out by the known method [1, 18, 19]. According to the standard control procedure we determine the value of the original number in PNS. In the end of the control it is determined that $A_{PNS} = 3360 > M = 420$. In this case, assuming the occurrence of only one-time (in one residue number) errors, it can be concluded that the considered number $\tilde{A}_{3360} = (0||0||0||0||5)$ is incorrect, i.e., one of the number residues is distorted. Then the procedure of determining AS $\tilde{A}_{3360} = (0||0||0||0||5)$ is realized (Table 1). For the number $A_{RNS} = (0 \parallel \; 0 \parallel \; 0 \parallel \; 0 \parallel \; 5)$ not distorted residues have been determined. They are $a_2 = 0$ and $a_3 = 0$. The values of residues on the bases m_1, m_4 and m_5, i.e., residues $a_1 = 0$, $a_4 = 0$ and $a_5 = 5$ may be incorrect. In this case, for the number $\tilde{A}_{RNS} = (0 \parallel 0 \parallel 0 \parallel 0 \parallel 5)$ AS will be equal to the set of RNS bases $W(\tilde{A}) = \{m_1, m_4, m_5\}$.

The use of the second method of data diagnostic in RNS allows us to speed up the process of determining AS $W(\tilde{A}) = \{m_{l_1}, m_{l_2}, \ldots, m_{l_\rho}\}$ of the number $\tilde{A}_{RNS} = (a_1||a_2||\ldots||a_{i-1}||\tilde{a}_i||a_{i+1}||\ldots||a_n||a_{n+1})$, due to the possibility of parallel determination of projections $\tilde{A}_j$ of incorrect number $\tilde{A}_{RNS} = (a_1||a_2||\ldots||a_{i-1}||\tilde{a}_i||a_{i+1}||\ldots||a_n||a_{n+1})$. It should be noted, that for the second method the procedure of determining the number of AS includes such basic operations as transferring $\tilde{A}_{RNS} = (a_1||a_2||\ldots||a_{i-1}||\tilde{a}_i||a_{i+1}||\ldots||a_n||a_{n+1})$ from RNS to PNS; converting projections $\tilde{A}_{i\ RNS} = (a_1||a_2||\ldots||a_{i-1}||a_{i+1}||\ldots||a_n||a_{n+1})$ of the incorrect number $\tilde{A}_{RNS}$ from RNS to PNS and the operation of comparing them with the value M. In RNS the listed operations refer to non-positional operations, the implementation of which is very consuming both in time and hardware.

The known methods of diagnosing in RNS have the common drawback, that is the low efficiency of data diagnostic. This reduces the effectiveness of RNS usage for rapid implementation of integer-valued operations.

The third recent designed method of data diagnosis is presented in [2, 7, 8]. Its usage allows increasing the efficiency of diagnosing in RNS. The essence of the developed method of improving the efficiency of diagnosing data in RNS is that AS $W(\tilde{A}) = \{m_{l_1}, m_{l_2}, \ldots, m_{l_\rho}\}$ of the number $\tilde{A}_{RNS}$ is determined not in the whole interval $[jM,\ (j+1)M)$, which contains the incorrect number $\tilde{A}_{RNS}$, but only in a small numerical interval $\Delta A^{(H)} = (\tilde{A}_{RNS} - \tilde{A}_{RNS}^{(H)}) < M$, where $\tilde{A}_{RNS}{}^{(H)} = (0 \parallel 0 \parallel \ldots \parallel 0 \parallel \gamma_{n+1})$ is a number reduced to zero in RNS. The essence of reducing to zero in RNS is to replace the original number $\tilde{A}_{RNS} = (a_1||a_2||\ldots||a_{i-1}||\tilde{a}_i||a_{i+1}||\ldots||a_n||a_{n+1})$ with the number $A_{RNS}^{(H)} = (0 \parallel 0 \parallel \ldots \parallel 0 \parallel \gamma_{n+1})$, by using a sequence of transformations, by which any intermediate number does not go beyond the working range $0 \div M-1$. zeroisation procedure can be implemented by various methods. The essence of all these methods is that some minimum $ZC^{(i)}$ numbers, so called zeroisatio constants (ZC), are sequentially subtracted from the initial number $\tilde{A}_{RNS} = (a_1||a_2||\ldots||a_{i-1}||\tilde{a}_i||a_{i+1}||\ldots||a_n||\ldots||a_{n+k})$ until the number $\tilde{A}_{RNS}$ is converted into the number $A_{RNS}^{(H)} = (0 \parallel 0 \parallel \ldots \parallel 0 \parallel \gamma_{n+1})$ and the value of the number $\tilde{A}_{RNS}$ does not go beyond the range $[0, M)$. Geometrically, zeroisation procedure corresponds to the offset of the original number $\tilde{A}_{RNS} = (a_1||a_2||\ldots||a_{i-1}||\tilde{a}_i||a_{i+1}||\ldots||a_n||a_{n+1})$ to the left edge jM of its numeric range $[jM,\ (j+1)M)$. Thus, to eliminate the redundancy of AS $W(\tilde{A}) = \{m_{l_1}, m_{l_2}, \ldots, m_{l_\rho}\}$, by reducing the interval range of the number $\tilde{A}_{RNS}$, the values $A_{RNS}^{(H)} = (0 \parallel 0 \parallel \ldots \parallel 0 \parallel \gamma_{n+1})$ and $\Delta A^{(H)} = (\tilde{A}_{RNS} - \tilde{A}_{RNS}^{(H)})\ mod\ M$ have to be predefined. It can be conveniently demonstrated for particular RNS.

As an example, for RNS defined by the bases $m_1 = 2,\ m_2 = 3,\ m_3 = m_{n+1} = 5$ $(M = 2 \cdot 3 = 6; M_0 = 2 \cdot 3 \cdot 5 = 30)$ (Table 2), in accordance with the distribution of errors in the intervals of the working range $[0, M)$ [1], for each interval $[jM,\ (j+1)M)$ two-entry tables are preliminarily compiled. Tables 3 of the correspondence of $\overline{W}(\tilde{A}) = \Phi(\gamma_{n+1};\ \ \Delta A^{(H)})$.

Table 2. Code Words in RNS

A to PNS	A in RNS			A to PNS	A in RNS		
0	0	0	0	15	1	0	0
1	1	1	1	16	0	1	1
2	0	2	2	17	1	2	2
3	1	0	3	18	0	0	3
4	0	1	4	19	1	1	4
5	1	2	0	20	0	2	0
6	0	0	1	21	1	0	1
7	1	1	2	22	0	1	2
8	0	2	3	23	1	2	3
9	1	0	4	24	0	0	4
10	0	1	0	25	1	1	0
11	1	2	1	26	0	2	1
12	0	0	2	27	1	0	2
13	1	1	3	28	0	1	3
14	0	2	4	29	1	2	4

As it was noted above AS $W(\tilde{A}) = \left\{m_{l_1}, m_{l_2}, \dots, m_{l_\rho}\right\}$ numbers are determined not on the whole range $[jM,\ (j+1)M)$, which contains the incorrect number $\tilde{A}$, but only on the numerical range $\Delta A^{(H)}$. The method of on-line data diagnostic in RNS is presented in Fig. 1.

Table 3. Table of values AS $\overline{W}(\tilde{A}) = \Phi(\gamma_{n+1};\ \ \Delta A^{(H)})$

$\Delta\tilde{A}$	γ_{n+1}			
	Z_1		Z_2	
	1	2	3	4

Z_3	0	m_3	m_2, m_3	m_1, m_3	m_2, m_3
	1	m_3	m_2, m_3	m_1, m_3	m_2, m_3
	2	m_3	m_2, m_3	m_1, m_2, m_3	m_3
Z_4	3	m_3	m_1, m_2, m_3	m_2, m_3	m_3
	4	m_2, m_3	m_1, m_3	m_2, m_3	m_3
	5	m_2, m_3	m_1, m_3	m_2, m_3	m_3

The considered method allows reducing the time of data diagnostic in RNS. The time to diagnose data is reduced, firstly, by eliminating non-positional operations such as converting numbers from RNS to PNS and comparing numbers, and, secondly, by using a single-entry tabular sampling of AS value. The proposed method of the rapid diagnostic of data errors improves the overall efficiency of using non-positional code structures in RNS.

The drawback of the considered method of rapid data diagnostics in RNS is the considerable amount of equipment required for its implementation due to the large volumes ($\Delta\tilde{A} \times (\gamma_{n+1} - 1)$ is a memory unit) of the memory (MMU) realizing function $\Phi(\gamma_{n+1};\ \Delta A^{(H)})$. We propose the following improvements in order to reduce the amount of the necessary equipment to implement the method of rapid diagnostic.

The essence of the improvements is to decrease in half the amount of the required equipment for the implementation of MMU content. This allows reducing the total amount of the required equipment for the implementation of the procedure for error diagnosing in NCS presented in RNS [20-22].

This is done by using the symmetry properties of the numerical data of the complete MMU table (Table 7) relative to the point with coordinate $\frac{M_0+M-1}{2}$, that corresponds to the value m_2, m_3 and is analytically expressed in the following way:

$$\overline{W}(\tilde{A}) = \Phi_1(\gamma_{n+1};\ \Delta A^{(H)}) = \Phi_2\{[m_{n+1} - \gamma_{n+1}]; [(M-1) - \Delta A^{(H)}]\} \qquad (2)$$

1. For a given RNS, a two-entry (two-coordinate) table of AS $\overline{W}(\tilde{A}) = \Phi(\gamma_{n+1};\ \Delta A^{(H)})$ values contained in MMU is compiled. There $1 \leq \gamma_{n+1} \leq m_{n+1} - 1$. Each pair of values γ_{n+1} and $\Delta A^{(H)}$ corresponds to a specific set of AS bases.

2. By means of a set of reduction to zero constants $ZC^{(i)}$ initial incorrect $\tilde{A}_{RNS} = (a_1||a_2||\ldots||a_{i-1}||\tilde{a}_i||a_{i+1}||\ldots||a_n||a_{n+1})$ number converted (reduced to zero) to $A^{(H)} = (0 \parallel 0 \parallel \ldots \parallel 0 \parallel \gamma_{n+1})$ number. We obtain value γ_{n+1} that corresponds to the first coordinate in the lookup table $\overline{W}(\tilde{A}) = \Phi(\gamma_{n+1};\ \ \Delta A^{(H)})$.

3. $\Delta A^{(H)} = \tilde{A}_{RNS} - \tilde{A}_{RNS}^{(H)}$ is determined. Therefore we obtain value $\Delta A^{(H)}$ of the second coordinate it the lookup table $\overline{W}(\tilde{A}) = \Phi(\gamma_{n+1};\ \ \Delta A^{(H)})$.

4. According to obtained values of two coordinates $\Delta A^{(H)}$ and γ_{n+1} we refer to the two-entry lookup table $\overline{W}(\tilde{A}) = \Phi(\gamma_{n+1};\ \ \Delta A^{(H)})$ from which the specific value of AS $W(\tilde{A}) = \{m_{l_1}, m_{l_2}, \ldots, m_{l_\rho}\}$ of incorrect number $\tilde{A}_{RNS} = (a_1||a_2||\ldots||a_{i-1}||\tilde{a}_i||a_{i+1}||\ldots||a_n||a_{n+1})$ in RNS is determined.

Fig. 1. Method of on-line data diagnostic in RNS

The correctness of (2) can be easily shown by using the results of the lemma on the distribution of the terms of number sequence $A_{is} = (a_1,\ a_2, \ldots, a_{i-1},\ s, a_{i+1}, \ldots,\ a_n,\ a_{n+1})$ in the numerical range $(0,\ M_0)$, where $s = 0,\ 1,\ \ldots,\ m_{i-1} (i = \overline{1,\ n+1})$ [1, 7, 8]. Basing on (2), the content of MMU for the proposed method of data diagnostic in RNS is presented in Table 4. Table 5 presents the characteristics Z_i of quadrant numbers from the completed Table 3 of MMU data and Table 6 presents the attributes of quadrant numbers of the shortened Table 4 of MMU data. In Table 7 there are the values of numerical ranges for finding the MMU input numbers and the correspondent data attributes formed by the group of decoders.

When implementing this method of data diagnostic in RNS [21, 22], in the diagnostic scheme the module of determining characteristics is intended for to form and use the characteristics $Z_1 \div Z_4$ of quadrant numbers $\Delta\tilde{A} \times (\gamma_{n+1} - 1)$ of the completed data table MMU $W(\tilde{A}) = \{m_{l_1}, m_{l_2}, \ldots, m_{l_\rho}\}$ (Table 3). The characteristics are formed by means of a group of decoders (Table 4) and a combination of OR elements. Using the values $Z_1 \div Z_4$, according to input data γ_{n+1} and $\Delta\tilde{A}$, the AS $W(\tilde{A}) = \{m_{l_1}, m_{l_2}, \ldots, m_{l_\rho}\}$ is determined by shortened table $\Delta\tilde{A} \times \left(\frac{\gamma_{n+1}-1}{2}\right)$ of MMU data (Table 4).

Table 4. AS $W(\tilde{A})$ values of shortened MMU

$\Delta\tilde{A}$	γ_{n+1}
	$Z_i Z_1$

		1	2
Z_3	0	m_3	m_2, m_3
	1	m_3	m_2, m_3
	2	m_3	m_2, m_3
Z_4	3	m_3	m_1, m_2, m_3
	4	m_2, m_3	m_1, m_3
	5	m_2, m_3	m_1, m_3

Table 5. Characteristics Z_i $(i = \overline{1,\ 4})$ of quadrant numbers $\Delta\tilde{A} \times (\gamma_{n+1} - 1)$ of the completed table AS data $W(\tilde{A}) = \{m_{l_1}, m_{l_2}, \ldots, m_{l_\rho}\}$

II (Z_1, Z_3)	I (Z_2, Z_3)
III (Z_1, Z_4)	IV (Z_2, Z_4)

Table 6. Characteristics of quadrant numbers $\Delta\tilde{A} \times \left(\frac{\gamma_{n+1}-1}{2}\right)$ of the table of the data $W(\tilde{A}) = \{m_{l_1}, m_{l_2}, \ldots, m_{l_\rho}\}$

II (Z_1, Z_3)
III (Z_1, Z_4)

The characteristics $Z_1 \div Z_4$ are applied as follows (Table 7): Z_1 and Z_2 are the characteristics of finding a distorted $\tilde{A}_{RNS}$ number in the numerical ranges $1 \div \frac{m_{n+1}-1}{2}$ and $\frac{m_{n+1}+1}{2} \div m_{n+1} - 1$ respectively; Z_3 and Z_4 - characteristics of finding a distorted number $\tilde{A}_{RNS}$ in the numerical ranges $0 \div \frac{(M-1)-1}{2}$ and $\frac{M}{2} \div M - 1$ respectively. For the second (II) and the third (III) quadrants, shortened Table 6, AS $W(\tilde{A})$ values are determined by formula $W(\tilde{A}) = F_1[\gamma_{n+1};\ \Delta A^{(H)}]$. For the first (I) and the fourth (IV) quadrants of the completed Table 3, according to the values of the shortened Table 4, AS $W(\tilde{A})$ values are determined by formula $W(\tilde{A}) = F_2\{[m_{n+1} - \gamma_{n+1}];\ \ [(M-1) - \Delta A]\}$.

Table 7. The value of numerical ranges and their correspondence to the data attributes

Decoder Group Outputs	Numerical range	Numerical range attribute
The group of the first decoder outputs (the first group of MMU inputs)	$1 \div \frac{m_{n+1}-1}{2}$	z_1
The group of the second decoder outputs (the second group of MMU inputs)	$0 \div M-1$	z_1, z_4
The first group of the third decoder outputs	$1 \div \frac{m_{n+1}-1}{2}$	z_1
The second group of the third decoder outputs	$\frac{m_{n+1}+1}{2} \div m_{n+1} - 1$	z_2
The first group of the fourth decoder outputs	$0 \div \frac{(M-1)-1}{2}$	z_3
The second group of the fourth decoder outputs	$\frac{M}{2} \div M-1$	z_4

The method of rapid data diagnostic in RNS is presented in Fig. 2

1. A two-entry (two-coordinate) table of AS $\overline{W}(\tilde{A}) = \Phi(\gamma_{n+1};\ \ \Delta A^{(H)})$ values of the MMU content is compiled where $1 \le \gamma_{n+1} \le \frac{m_{n+1}-1}{2}$.Each pair of values γ_{n+1} and $\Delta A^{(H)}$ corresponds to a specific set of AS bases.

2. By means of a set of reduction to zero constants $ZC^{(i)}$ initial incorrect $\tilde{A}_{RNS} = (a_1||a_2||\ldots||a_{i-1}||\tilde{a}_i||a_{i+1}||\ldots||a_n||a_{n+1})$ number is converted (reduced to zero) into the following $A_{RNS}^{(H)} = (0 \parallel 0 \parallel \ldots \parallel 0 \parallel \gamma_{n+1})$ number.

3. The analysis of the magnitude of obtained γ_{n+1} value. If the condition $1 \le \gamma_{n+1} \le (m_{n+1}-1)/2$ is not met, i.e. $\gamma_{n+1} > \frac{m_{n+1}-1}{2}$ then the subtraction $(m_{n+1} - \gamma_{n+1})\,mod\,m_{n+1}$ is performed. The value of $(m_{n+1} - \gamma_{n+1})\,mod\,m_{n+1}$ is the first coordinate of $\overline{W}(\tilde{A}) = \Phi(\gamma_{n+1};\ \ \Delta A^{(H)})$ table.

4. $\Delta A^{(H)} = \tilde{A}_{RNS} - \tilde{A}_{RNS}^{(H)}$ is determined. Therefore we obtain value $\Delta A^{(H)}$ of the second coordinate it the lookup table $\overline{W}(\tilde{A}) = \Phi(\gamma_{n+1};\ \ \Delta A^{(H)})$.

5. According to obtained values of two coordinates $\Delta A^{(H)}$ and γ_{n+1} we refer to the two-entry lookup table $\overline{W}(\tilde{A}) = \Phi(\gamma_{n+1};\ \ \Delta A^{(H)})$ from which the specific value of AS $W(\tilde{A}) = \left\{m_{l_1}, m_{l_2}, \ldots, m_{l_\rho}\right\}$ of incorrect number $\tilde{A}_{RNS} = (a_1||a_2||\ldots||a_{i-1}||\tilde{a}_i||a_{i+1}||\ldots||a_n||a_{n+1})$ in RNS is determined.

Fig. 2. Method of on-line data diagnostic in RNS.

2 Examples of using the method of rapid data diagnostic in RNS

In accordance with Fig. 2, let us present the examples 2-4 [21, 22] of using the method of on-line data diagnostic in RNS determined by bases $m_1 = 2$, $m_2 = 3$, $m_3 = m_{n+1} = 5$; $M = 2 \cdot 3 = 6$; $M_0 = 2 \cdot 3 \cdot 5 = 30$ (Table 2). Tables 8 and 9 present some zeroisation constants for the corresponding RNS basis.

Table 8. The reduction to zero constants for the first base of RNS

a_1	ZC
0	(0 ‖ 0 ‖ 0)
1	(1 ‖ 1 ‖ 1)

To check the obtained diagnostic result $W(\tilde{A}) = F_{RES}[\gamma_{n+1}; \Delta\tilde{A}]$, which is determined by the shortened Table 4 of $W(\tilde{A})$ MMU of the dimension $\Delta\tilde{A} \times \left(\frac{\gamma_{n+1}-1}{2}\right)$, the values $W(\tilde{A}) = F_{TEST}[\gamma_{n+1}; \Delta\tilde{A}]$ are used, which are determined by the completed Table 3 of MMU data of the dimension $\Delta\tilde{A} \times (\gamma_{n+1} - 1)$.

Table 9. The reduction to zero constants for the second base of RNS

a_2	ZC
0	(0 ‖ 0 ‖ 0)

1	$(0 \parallel 1 \parallel 4)$
2	$(0 \parallel 2 \parallel 2)$

Example 2. To diagnose the number $\tilde{A} = (0 \parallel 1 \parallel 2)$ i.e. to identify AS $W(\tilde{A})$ of the number $\tilde{A} = (0 \parallel 1 \parallel 2)$. The value of the zeroisation number is represented as $\tilde{A}^{(H)} = \tilde{A} - KH = (0 \parallel 1 \parallel 2) - (0 \parallel 1 \parallel 4) = (0 \parallel 0 \parallel 3)$ (Table 9). As $\gamma_{n+1} \neq 0$ $(\gamma_3 = 3)$, therefore the number $\tilde{A} = (0 \parallel 1 \parallel 2)$ was distorted in one of the residues. The value $\gamma_{n+1} = 3$ (in binary code 011) is fed to the input of the third decoder, from the output of which the value $\gamma_{n+1} = 3$ is fed to the input of the OR element in the unitary code (Table 7). Then, the value $\gamma_{n+1} = 3$ in binary code is fed to the inverter's inputs by modulo 5, from the outputs of which the value $m_{n+1} - \gamma_{n+1} = 5 - 3 = 2$ (in binary code 010) is fed to the input of the first decoder, from the output of which the value 2 is fed to the second input of the first group of MMU inputs in the unitary code (Tables 4, 7).

The value $\Delta\tilde{A} = \tilde{A} - \tilde{A}^{(H)} = (0 \parallel 1 \parallel 2) - (0 \parallel 0 \parallel 3) = (0 \parallel 1 \parallel 4)$ (in binary code 100) is fed to the input of the fourth decoder, from the output of which the value $\Delta\tilde{A} = 4$ is fed to the input of the corresponding OR element in the unitary code (Table 7). Simultaneously, the value $\Delta\tilde{A} = 4$ (in the binary code 100) is fed to the inputs of the inverter and on the output of which the value $(M-1) - \Delta\tilde{A} = (6-1) - 4 = 1$ (in binary code 001) is fed to the input of the decoder, from the output of which the value 1 in the unitary code is fed to the first input of the second group of MMU inputs (Tables 4, 7).

According to the data $W(\tilde{A})$ of MMU table (Table 4), by value γ_{n+1} which equals two and the value $\Delta\tilde{A}$ which equals one, we receive $W(\tilde{A}) = \{m_2, m_3\}$ as the result of the procedure on the output of the device. Thus the result of diagnosing the number $\tilde{A} = (0 \parallel 1 \parallel 2)$ is obtained in the following way: $W(\tilde{A}) = F_{RES}\big[(m_{n+1} - \gamma_{n+1}); [(M-1) - \Delta\tilde{A}]\big] = F_{RES}(2;\ 1) = \{m_2, m_3\}$.

Check (Table 3): $W(\tilde{A}) = F_{TEST}(\gamma_{n+1}; \Delta\tilde{A}) = F_{TEST}(3;\ 4) = \{m_2, m_3\}$.

Example 3. It is assumed to determine AS $W(\tilde{A})$ of the number $\tilde{A} = (1 \parallel 1 \parallel 2)$. The value of the zeroisaton number is represented as $\tilde{A}^{(H)} = \tilde{A} - KH = (1 \parallel 1 \parallel 2) - (1 \parallel 1 \parallel 1) = (0 \parallel 0 \parallel 1)$ (Table 8). Thus, we have the value $\gamma_{n+1} = 1$ (in binary code 001) and also determine that $\Delta\tilde{A}^{(H)} = \tilde{A} - \tilde{A}^{(H)} = (1 \parallel 1 \parallel 2) - (0 \parallel 0 \parallel 1) = (1 \parallel 1 \parallel 1)$. The value $\gamma_{n+1} = 1$ (in binary code 001) is fed to the input of the decoder, from the output of which the value $\gamma_{n+1} = 1$ is fed to the input of the corresponding element OR in the unitary code. The value $\Delta\tilde{A}^{(H)} = 1$ (in binary code 001) is fed to the input of the fourth decoder, from the output of which the value $\Delta\tilde{A}^{(H)} = 1$ is fed to the input of the corresponding OR element in the unitary code (Table 7). The value $\gamma_{n+1} = 1$ (in the binary code 001) is fed to the decoder, from the output of which the value 1 in a unitary code, through a corresponding OR element, is fed to the first input of the first groups of MMU inputs. At the same time, the value $\Delta\tilde{A}^{(H)} = 1$ (in binary code 001) is fed to the input of the second decoder, from the output of which value 1 in the unitary code is fed to the first input of the second group of MMU inputs (Table 4). In accordance with the $W(\tilde{A})$ data of MMU (Table 4), we obtain $W(\tilde{A}) = \{m_3\}$ as the result of the procedure. Therefore $W(\tilde{A}) = F_{RES}(\gamma_{n+1};\ \Delta\tilde{A}) = F_{RES}(1;\ 1) = \{m_3\}$.

Check (Table 3): $W(\tilde{A}) = F_{TEST}(\gamma_{n+1}; \Delta\tilde{A}) = F_{TEST}(1;\ 1) = \{m_3\}$.

Example 4. Number $\tilde{A} = (0 \parallel 0 \parallel 4)$ is assumed to be diagnosed (AS $W(\tilde{A})$ of $\tilde{A} = (0 \parallel 0 \parallel 4)$ number must be determined). First $\gamma_{n+1} = 4 \neq 0$ is determined. Then we obtain $\Delta\tilde{A} = \tilde{A} - \tilde{A}^{(H)} = (0 \parallel 0 \parallel 4) - (0 \parallel 0 \parallel 4) = (0 \parallel 0 \parallel 0)$ and therefore $\Delta\tilde{A} = 0$. Value $\gamma_{n+1} = 4$ is fed to the input of the decoder, from the output of which value $\gamma_{n+1} = 4$ is fed to the input of the OR element in the unitary code (Table 7). The value $\Delta\tilde{A} = 0$ is fed to the input of the fourth decoder, from the output of which value $\Delta\tilde{A} = 0$ is fed to the input of the OR element in the unitary code. The value $\gamma_{n+1} = 4$ (in the binary code 100) is fed to the inverter from the output of which the value $m_{n+1} - \gamma_{n+1} = 5 - 4 = 1$ (in the binary code 001) is fed to the first decoder from the output of which the value 1 in a unitary code, through the corresponding OR element, is fed to a first input of the first group of MMU inputs (Table 4). Simultaneously, the value $\Delta\tilde{A} = 0$ is fed to the inverter in binary cod, from the output of which the value $(\mathrm{M} - 1) - \Delta\tilde{A} = (6 - 1) - 0 = 5$ (in the binary code 101), through the OR element, is fed to the decoder input from the output of which the value 5 is fed to the fifth input of the second group of MMU inputs in a unitary code (Table 4). In accordance with the $W(\tilde{A})$ data of MMU (Table 4), the result of the diagnosing is determined by the value γ_{n+1} that equals 1, and by the value $\Delta\tilde{A}$ that equals 5. Therefore the procedure result is

$W(\tilde{A}) = \{m_2, m_3\}$=$W(\tilde{A}) = F_{RES}\big[(m_{n+1} - \gamma_{n+1}); [(M - 1) - \Delta\tilde{A}]\big] = F_{RES}(1;\ 5) = \{m_2, m_3\}$.

Check (Table 3): $W(\tilde{A}) = F_{TEST}(\gamma_{n+1};\ \Delta\tilde{A}) = F_{TEST}(4;\ 0) = \{m_2, m_3\}$.

3 Conclusion

According to the results of studying the methods of data diagnostic in RNS the improved method of rapid diagnostic is proposed for the practical implementation. Application of this method allows reducing the amount of the equipment required for implementing data diagnostic procedures in RNS without increasing the time of diagnosis. This is achieved by reducing the amount of equipment for completed table $\Delta\tilde{A} \times (\gamma_{n+1} - 1)$ of MMU, by forming and using numerical characteristics $Z_1 \div Z_4$ which show the belonging of the input numbers γ_{n+1} and $\Delta\tilde{A}$ of the table of MMU to each of the four quadrants of the completed data table AS $W(\tilde{A})$ of the numbers $\tilde{A}$ in RNS. This makes it possible to perform reliable diagnostic of the distorted number $\tilde{A}$ in RNS, i.e., precisely determine those bases of RNS where the residues of the correct number *A* have been distorted. The values of only a half (the second and the third quadrants) of the completed data table AS $W(\tilde{A})$ of MMU are used. The examples of the practical usage of the method of diagnosis have been presented. The verification of the diagnosis of numbers in RNS, carried out by the developed method confirms the validity of the stated goal and the practical feasibility of diagnosing data in RNS. Based on the proposed diagnostic method, an algorithm of its implementation has been developed and the patentable device has been produced. A device for monitoring and diagnosing data presented in RNS has been patented in Ukraine. It should be noted that by increasing the length of the discharge grid of the calculator in RNS, the efficiency of the proposed method also increases.

The article has obtained some theoretical results that make an academic contribution to the theory of creating computer systems functioning in RNS. The theoretical issues of creating a system of correction of CS data in RNS, based on the results obtained in this article, are not considered. This is a certain limitation in the creation of a further correction system for CS data. To overcome this limitation, based on the scientific results obtained in this article, it is necessary to conduct additional fundamental research in the field of error correction in RNS.

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