Mathematical Modeling of the Working Body's Oscillatory Motion in a Concrete Mixer

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Abstract. This study investigates the concrete mixing process in a gravity mixer and proposes strategies to enhance its efficiency. The challenge of uneven component distribution, arising from passive-zone formation, was addressed through a mathematical model developed to describe particle-motion kinematics within the mixing drum. Numerical simulations demonstrated that mixing efficiency is strongly influenced by drum rotation speed, inclination angle, blade configuration, and oscillatory motion. Introducing oscillations was found to increase the intensification coefficient by 15–18 %, reduce the passive-zone area from 28 % to 15 %, and improve mixture uniformity by 12–15 % in terms of the variation coefficient. Furthermore, oscillatory motion accelerates the growth of homogeneity: a rapid increase begins as early as 2 min, reaching the mixing intensity factor 0.8 by 4 min, corresponding to high-quality mixing. In contrast, without oscillations, a comparable level of homogenization is achieved only after 6–7 min of drum operation. The findings confirmed the effectiveness of oscillatory drum motion as a practical approach to improving mixing quality, reducing energy demand, and optimizing the structural design of concrete mixers.

Keywords: concrete mixture homogenization, drum-type gravity mixer, granular flow dynamics, active and passive zones, mixing efficiency optimization, numerical simulation of particle motion, harmonic oscillation of mixing drum.

1 Introduction

Modern construction requires high-quality building materials that ensure strength, durability, and efficiency. One of the most critical components in the construction industry is concrete, which is used in various fields – from residential construction to industrial and infrastructure facilities. The quality of concrete largely depends on its uniformity, which is achieved by carefully mixing the components. That is why optimizing the process of mixing concrete mixtures is becoming particularly relevant, as it allows you to increase the material's operational characteristics and reduce production energy costs [1].

Mixing a concrete mixture is a complex physical and mechanical process that involves the movement, interaction, and distribution of particles in the working space of the mixing device. Various types of concrete mixers are used to ensure uniform distribution of the mixture components, among which gravity concrete mixers occupy a significant place. They ensure particle mixing under the influence of gravitational, centrifugal, and frictional forces, allowing the required mixture quality

to be achieved. At the same time, the efficiency of such mixing largely depends on the mixer's design parameters, particularly the drum's rotation frequency, inclination angle, and blade geometry [2].

The study of the kinematic and dynamic characteristics of particle movement in a concrete mixer is essential for describing the basic patterns of the process and determining the optimal mixing conditions. Traditionally, mixing effectiveness is estimated by experimental methods, but mathematical modeling and computer simulations have recently become increasingly widespread. They make it possible not only to predict the mixture's behavior in different operating modes of the mixer but also to optimize its design without conducting numerous costly experiments [3].

One of the key factors influencing mixing quality is the distribution of particles between active and passive zones within the concrete mixer drum. Active zones exhibit intensive particle movement, whereas passive zones lead to particle accumulation and the formation of so-called "dead zones". Uneven mixing can reduce the final concrete mix quality, create inhomogeneities in its structure, and

weaken its strength. Therefore, minimizing passive zones and ensuring a uniform distribution of components throughout the mixing drum's volume is crucial [4].

Applying an oscillatory motion mode to the working body can improve the mixing process, significantly enhancing mixing efficiency by altering particle motion trajectories. Additional oscillatory movements expand active zones while reducing passive zones, promoting a more uniform material distribution. However, such changes in particle motion kinematics require careful analysis and mathematical modeling, as their influence on the mixing process depends on numerous parameters, including the mixer's geometry, material properties, and operating modes.

This study focuses on optimizing the concrete mixing process in a gravity mixer by analyzing particle motion trajectories, identifying active and passive zones in the drum, and evaluating the effectiveness of oscillatory motion. Mathematical modeling and theoretical analysis are employed to achieve these goals.

The results will improve the design of concrete mixers, increase their efficiency, and improve the quality of the final product. This, in turn, will contribute to reducing production and energy costs and increasing the competitiveness of modern technologies for mixing concrete mixtures. Understanding the basic patterns of particle kinematics in a mixer is essential to creating more efficient and cost-effective mixing approaches with a wide range of applications in the construction industry.

2 Literature Review

Concrete mixing is a crucial stage in building material production, and it has attracted significant research attention over the past decades. Various approaches to mixing analysis and mixer design optimization have been extensively studied in scientific literature, covering experimental, numerical, and theoretical methods for evaluating mixing efficiency. Research primarily focuses on the kinematic analysis of particle movement and the improvement of their displacement process [5–7].

The study of concrete mixture mixing has been conducted by classical researchers in fluid mechanics and granular media, as well as by modern authors employing advanced computer modeling and machine learning techniques. Significant contributions to mixing theory have been made by researchers investigating particle movement mechanisms in various mixing devices [8–10]. One of the most influential approaches to process analysis involves applying fluid mechanics and turbulence theory equations, which describe the motion of mixture components within the working volume of the concrete mixer [11].

Gravity concrete mixers are among the most widely used equipment for preparing concrete mixtures, making their kinematics and dynamics a key focus of many scientific studies. Several works have explored the impact of drum design parameters – such as diameter, length, shape, and blade arrangement – on mixing efficiency. Research has shown that an optimal blade configuration can significantly enhance mixing quality by minimizing passive zones where particles remain stationary [12, 13].

The literature emphasizes the impact of drum rotation frequency and inclination angle on mixing efficiency. Studies indicate that a low rotation frequency results in insufficient particle movement, leading to uneven distribution and lump formation. Conversely, an excessively high frequency can cause excessive centrifugal displacement of particles toward the drum walls, reducing mixing efficiency and potentially causing mixture stratification. Optimizing these parameters is crucial for achieving maximum material homogeneity [14, 15].

Based on the analysis in scientific literature, it can be concluded that the efficiency of concrete mixing largely depends on the design parameters, kinematic characteristics of particle motion, and features of hydrodynamic processes within the working volume. The most promising research areas are further developing mathematical modeling, applying design optimization methods, and introducing new technologies to automate and enhance the process's efficiency. An expanded study of particle motion kinematics and active zones within the mixer will allow the development of more effective design solutions, ensuring the homogeneity of the concrete mixture and reducing energy costs.

3 Research Methodology

The efficiency of the concrete mixing process largely depends on the kinematic characteristics of particles in the working space within the concrete mixer drum. Determining optimal mixing parameters requires a detailed analysis of the mixture's particle trajectories, velocity distribution, and dynamic characteristics.

Particle motion characteristics, component distribution uniformity, and the kinematic characteristics of the mixture flow determine the process of mixing concrete in a gravity concrete mixer. Optimizing these parameters is key to ensuring the final product's high quality, as the mixture's homogeneity and technological properties depend on it.

The main element in a gravity concrete mixer (Figure 1) is a drum that rotates around its axis at a certain angle.
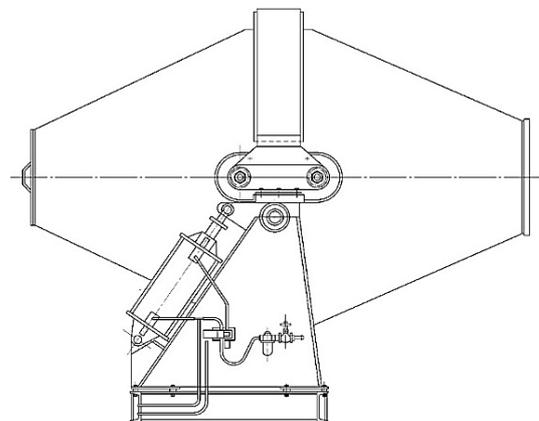

Figure 1 – Gravity concrete mixer

Its design determines the dynamics of mixture particle movement, particularly their motion under the influence of gravity, centrifugal forces, and friction. The blade configuration inside the drum is necessary, contributing to active mixing and the uniform distribution of components [16].

General mixing ensures the blending of large volumes of concrete mix within the working space of the concrete mixer drum. The effectiveness of this process depends on particle motion trajectories, component distribution uniformity, the mix dynamic characteristics, and the equipment's design features [17].

The equation can describe the particle trajectory determination:

$$\vec{r}(t) = \vec{r}_0 + \int_0^t \vec{v}(t)dt, \qquad (1)$$

where t – time, s; $\vec{r}_0$, $\vec{r}$ – the initial and current positions of the particle, respectively, m; $\vec{v}(t)$ – the velocity, m/s.

The gravitational velocity component accounts for the particle's vertical motion under gravity. If the initial vertical velocity is zero, its velocity is determined by the following equation:

$$\vec{v}_{grav}(t) = \int \vec{g}\,dt = \vec{v}_0 + \vec{g}t, \qquad (2)$$

where $\vec{v}_0$ – the initial velocity, m/s; $\vec{g}$ – the gravity acceleration, m/s^2.

Since the particle moves in a variable velocity field within the mixer, gravitational acceleration plays a key role in shaping its trajectory. The particle's motion may involve intermittent velocity changes due to interactions with other particles or the drum walls, affecting mixing uniformity.

A particle's movement in the drum of a concrete mixer occurs under the influence of centrifugal force, arising from the drum's rotational motion. Centrifugal force compels the particle to move along a curved trajectory, keeping it near the inner surface of the mixer. However, the main parameter determining the particle's movement is not the force itself, but its speed.

Centrifugal velocity depends on the drum's angular velocity and the radius at which the particle is located [18]. As angular velocity increases, the particle's linear velocity also increases, which can lead to material stratification. In this case, heavier components shift to the periphery, while lighter ones remain closer to the center:

$$\vec{v}_{centr}(t) = \int \frac{\vec{F}_{centr}}{m}dt = \omega^2 R \vec{e}_r, \qquad (3)$$

where $\vec{F}_{centr}$ – the centrifugal force, N; m – the particle mass, kg; ω – the drum's angular rotation speed, rad/s; R – the radius at which the particle is located, m; $\vec{e}_r$ – the unit vector in the radial direction.

In addition to centrifugal forces, the particle is affected by the frictional force, which determines its relative motion relative to the inner surface of the drum and other particles in the mixture. Friction is a key factor contributing to the change in the speed of particles and their mixing.

The velocity caused by friction depends on the magnitude of the normal force acting on the particle and the friction coefficient. If friction is large enough, the particle can move with the drum surface, changing its trajectory due to interactions with other particles. Conversely, the particle can easily slip if the friction is too small, reducing mixing intensity.

To fully describe the kinematics of particle motion in a mixer, it is necessary to account for the interaction between gravitational, centrifugal, and frictional forces. Since particle motion occurs under the simultaneous influence of these forces, the total change in its velocity can be expressed as the derivative of velocity with respect to time. This equation allows estimation of how particle velocity changes over time, considering all the main physical factors [19, 20]:

$$\frac{dv}{dt} = g + \frac{\vec{F}_{centr}}{m} - \mu N, \qquad (4)$$

where μ – the coefficient of friction; N – the normal force, N.

Analysis of passive and active zones allows for optimizing the concrete mixer drum design, reducing energy costs, and ensuring the high quality of the final product. Active zones are areas where mixture particles actively move, interacting with each other and the drum blades. In contrast, passive zones are characterized by low particle movement intensity or their stationarity, creating "dead zones" and reducing mixing efficiency.

The velocity of particles in a mixer varies depending on their position and interaction with the material flow. A velocity gradient is used to assess which areas of the mixer are conducive to effective mixing and which are passive zones. This allows us to determine where there is intense particle movement and where stagnant areas occur that can negatively affect the homogeneity of the mixture:

$$\nabla v = \frac{\Delta v}{\Delta h}, \qquad (5)$$

where ∇v – the velocity gradient, s^{-1}; Δv – the change in the speed of particles in space, m/s; Δh – the distance between the layers of the mixture flow, m.

Passive zones are often located in the drum's central part, away from the blades, and in the upper section, where particles remain almost static (Figure 2).
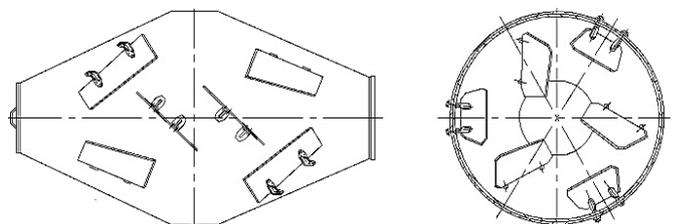

Figure 2 – Layout of the blades in the mixing drum

In such areas, the velocity gradient is minimal.

The equation can describe the influence of gravity and friction on particle motion:

$$\vec{F}_{grav} + \vec{F}_{fr} = 0, \qquad (6)$$

where $\vec{F}_{grav}$, $\vec{F}_{fr}$ – gravity and friction forces, respectively, N/m.

A generalized scheme illustrating the interaction of forces governing the motion of the concrete mixture inside the mixing drum is presented in Figure 3.
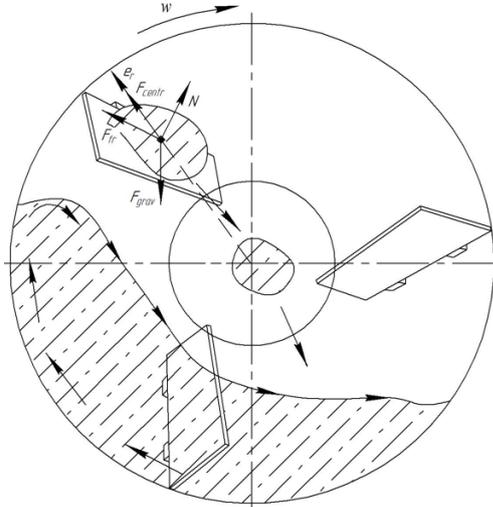

Figure 3 – Forces acting on the concrete mixture in the drum: w – angular velocity, rad/s; $\vec{F}_{centr}$, $\vec{N}$ – centrifugal and normal reaction forces, respectively, N

It depicts the action of gravity, centrifugal force, friction, and the support reaction. This visualization clarifies the relationship between the analytical expressions provided in the equations and the actual conditions of mixture motion.

A new approach to intensifying active zones by periodically changing the drum's inclination angle was proposed (Figure 4).
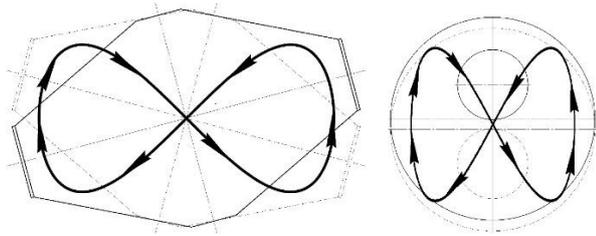

Figure 4 – Method of mixing the mixture

The introduced oscillatory component in the drum's motion promotes particle transfer from passive to active zones. It generates additional horizontal movement, ensuring uniform distribution throughout the drum's volume.

As a result, mixing in a gravity concrete mixer using this method exhibits complex particle dynamics, intensified by periodic inclination changes, which enhance component mixing uniformity and improve mixture homogeneity.

The drum's oscillatory motion plays a key role in enhancing mixing efficiency. Periodic inclination changes help prevent passive zone formation and ensure a more uniform particle distribution in the mixer. Oscillation promotes additional material movement in the horizontal direction, improving mixing efficiency and reducing the risk of mixture stratification.

A harmonic function describes the drum's oscillatory motion, accounting for inclination angle changes over time. This approach mathematically formalizes the impact of periodic oscillations on particle behavior within the mixing space. The oscillation amplitude defines the drum's maximum angular deviation from its initial position, while the frequency directly influences mixing dynamics.

The main parameter that determines the oscillations is the angle of inclination of the drum $\theta(t)$, which varies according to the harmonic law:

$$\theta(t) = \theta_0 \sin(\omega_{osc} t), \qquad (7)$$

where θ_0 – the maximum angle of inclination, rad; ω_{osc} – the frequency of oscillations, rad/s.

Forces arising during oscillatory motion can be divided into several components. Periodic acceleration changes influence particle trajectories within the mixing drum, generating additional impulses that enhance mixing efficiency and minimize passive zone formation. In the horizontal plane, these forces are described by the equations:

$$F_x = mR\frac{d^2 \sin(\theta(t))}{dt^2}; \; F_y = mR\frac{d^2 \cos(\theta(t))}{dt^2}, \qquad (8)$$

where F_x, F_y – projections of the force to the x- and y-axes, respectively, N.

A particle's movement in a mixer is shaped by the complex interaction of multiple forces influencing its kinematics. In the vertical direction, gravity remains the primary factor, complemented by centrifugal forces. Gravity ensures constant downward acceleration of particles, while centrifugal force, generated by the drum's rotation, acts radially, guiding particles along a curvilinear trajectory [21]. The interplay of these forces defines particle kinematics in the mixer and influences the uniform distribution of particles within the mixture. Under real conditions, vertical motion is further complicated by friction and possible impacts against the drum walls and other particles. Considering these interactions, the resulting force along the z-axis can be expressed as follows:

$$F_z = -mg + m\omega^2 R \sin(\theta(t)), \qquad (9)$$

where F_z – projection of the force to the z-axis, N.

The mixing intensity factor κ is introduced to evaluate the efficiency of oscillations, which determines how much oscillations improve the movement of particles compared to their absence:

$$\kappa = \frac{\vec{v}_{osc}}{\vec{v}_{without\ osc}}, \qquad (10)$$

where $\vec{v}_{osc}$ – the particle velocity taking into account oscillations, m/s; $\vec{v}_{without\ osc}$ – the particle velocity without oscillations, m/s.

This coefficient quantitatively assesses the drum's oscillatory motion impact on particle kinematics, facilitating a comparative analysis of different mixing modes. Higher κ values indicate greater mixture homogeneity and more effective particle attraction to the active mixing zone. However, an excessive oscillation amplitude or frequency increase may raise energy consumption without significantly improving mixing efficiency. Therefore, determining the optimal κ value is crucial for balancing process efficiency and energy use.

The dynamics of oscillatory motion also influence particle trajectories as they cross various mixture layers, promoting a more efficient spatial distribution of components. To describe these trajectories in 3D space with coordinates (x, y, z), the following equation is used:

$$\vec{r}(t) = \begin{Bmatrix} R \sin(\theta(t)) \cos(\omega t) \\ R \sin(\theta(t)) \sin(\omega t) \\ z_0 + v_{osc\,z}(t) \cdot t \end{Bmatrix}, \qquad (11)$$

where z_0 – the initial particle height, m; $v_{osc\,z}(t)$ – projection of the oscillation velocity $\vec{v}_{osc}$ to the z-axis, m/s.

In addition, friction between the mixture particles and the drum surface plays a significant role. This friction can either slow down particle movement or help attract it to the active mixing zones. Additionally, if the mixer drum has an oscillating motion, alternating forces arise that force particles to change their trajectory and become more evenly distributed in the working volume of the mixer.

Thus, an equation of motion based on Newton's second law is used to describe particle dynamics. This equation summarizes all external forces acting on the particle, allowing one to estimate changes in its velocity and position in space. Each equation component reflects the influence of the corresponding physical factor: gravity, centrifugal force, friction, and oscillatory vibrations. Therefore, the dynamics of particle motion in a mixer are described by the following equation:

$$m\frac{d^2\vec{r}}{dt^2} = \vec{F}_{grav} + \vec{F}_{centr} + \vec{F}_{fr} + \vec{F}_{osc}, \qquad (12)$$

where $\vec{F}_{osc}$ – oscillation force, N.

The trajectory of a particle is determined by integrating the motion equation for each coordinate x, y, z:

$$\begin{cases} x(t) = x_0 + \int_0^t v_x(t)dt, \\ y(t) = y_0 + \int_0^t v_y(t)dt, \\ z(t) = z_0 + \int_0^t v_z(t)dt, \end{cases} \qquad (13)$$

where x_0, y_0, z_0 – the initial coordinates of the particle, m; v_x, v_y, v_z – components of the particle velocity at time t, m/s.

The vertical motion of particles is determined by gravity's force and the medium's resistance:

$$m\frac{d^2z}{dt^2} = -mg + \mu\frac{dz}{dt}, \qquad (14)$$

where μ – the mixture damping coefficient, N·s/m.

For the equilibrium state, the vertical velocity of the particle is determined by the balance between gravitational force and medium resistance. The resistance depends on the viscous properties of the concrete mixture and its density, affecting the particles' final velocity. When equilibrium is reached, the gravitational force is compensated by resistance forces, and particle velocity stops changing, acquiring a stationary value. This state is essential for assessing the kinematics of material movement in the drum and allows for determining the critical mixing parameters:

$$v_z = \frac{mg}{\mu}. \qquad (15)$$

A complete system of equations is formulated to describe the mixture motion in the drum. A position vector $\vec{r}(t) = (x(t), y(t), z(t))$ defines each particle that satisfies the equations:

$$\begin{cases} m\frac{d^2x}{dt^2} = -m\omega^2 R \cos(\omega t) + mR\frac{d^2\sin(\theta(t))}{dt^2}, \\ m\frac{d^2y}{dt^2} = m\omega^2 R \sin(\omega t) + mR\frac{d^2\cos(\theta(t))}{dt^2}, \\ m\frac{d^2z}{dt^2} = -mg - \mu\frac{dz}{dt}. \end{cases} \qquad (16)$$

Boundary conditions are formulated for the boundaries of the drum. The particles must remain inside the drum, so the following radius limits their radius-vector:

$$x^2 + y^2 \leq R^2,$$

where R – the radius, m.

In addition, the movement of particles is limited by the geometry of the vanes, which is taken into account due to local boundary conditions.

In the mixing process, velocity distribution in the drum's working volume is essential, as it determines the mixture's homogeneity. Uneven particle motion can cause zones with different component concentrations, negatively affecting the quality of the final product. The diffusion equation is used to model the velocity field, allowing us to estimate how particle concentration changes in space based on the mixer's kinematic parameters and the mixture's rheological properties:

$$\frac{\partial v}{\partial t} = D\nabla^2 v, \qquad (17)$$

where D – the diffusion coefficient, m²/s; ∇^2 – the Laplace operator that determines the spatial change in concentration, m⁻²; v – the volumetric concentration of particles, m⁻³.

The homogeneity of a concrete mix is one of the key parameters determining its quality. Uneven distribution of components in the drum's working volume can lead to forming zones with different mechanical properties, negatively affecting the final concrete characteristics [22]. Therefore, assessing the degree of mix homogeneity is crucial for determining the efficiency of the mixing process.

The average particle velocity in the drum is a key parameter determining mixing efficiency. It depends on centrifugal and oscillatory forces that affect the particle's trajectory and acceleration. High average velocity values indicate active mixing, while low values may signal the

formation of passive zones where particles remain immobile. The following relationship is used:

$$\bar{v} = \frac{1}{T}\int_0^T v\,(t)dt, \qquad (18)$$

where T – the drum rotation period, s; $v(t)$ – the instantaneous velocity of the particle, m/s.

The obtained mathematical relationships allow for the estimation of the kinematics of particle motion in the working space within the concrete mixer drum, taking into account gravitational, centrifugal, and friction forces and the influence of the oscillatory motion of the drum. It was established that the rotation frequency, blade geometry, and drum inclination angle significantly affect the mixing efficiency. The proposed mathematical model allows for estimating the distribution of active and passive mixing zones and identifying the optimal mixer operation parameters to minimize the unevenness in mixing.

Mathematical modeling allows us to assess the influence of the drum's oscillatory motion on the kinematics of mixing. The harmonic component contributes to a more active movement of particles in both the vertical and horizontal directions, improving material distribution uniformity. The obtained analytical relationships can be used to optimize the concrete mixer design further and select rational operation parameters. Thus, the mathematical modeling allows us to assess the kinematics of particle movement in the mixing drum, taking into account gravitational, centrifugal, and friction forces and the influence of oscillatory motion. The obtained mathematical relationships make it possible to determine the main mixing process parameters and their impact on mixture homogeneity.

4 Results

A theoretical analysis assessed the proposed mathematical model's effectiveness, establishing the relationship between the concrete mixer's design parameters and the mixing uniformity. The main emphasis is studying particle motion's kinematics, determining their speeds and movement trajectories, and identifying active and passive mixing zones.

To assess mixing intensification effectiveness, the following volumetric efficiency indicator is provided, chosen based on the drum volume involved in the active mixing process:

$$\eta_{volume} = \frac{V_{mixing}}{V_{drum}}, \qquad (19)$$

where η_{volume} – the volumetric efficiency indicator; V_{mixing} – the mixing core volume, m³; V_{drum} – the total drum volume, m³.

The turbulent flow caused by oscillations significantly increases mixing efficiency by creating additional contact points between particles. To assess turbulence, the Reynolds number is used, characterizing the ratio of inertial to viscous forces, and can be calculated as:

$$Re = \frac{\rho v L}{\mu_0}, \qquad (20)$$

where ρ – the mixture density, kg/m³; v – the average velocity of particles, m/s; L – the hydraulic diameter as a characteristic drum size, m; μ – the dynamic viscosity, Pa·s.

The blades located at an angle to the radial direction contribute to additional shear deformations in the mixture, described by the velocity gradient:

$$\dot{\gamma} = \frac{\Delta v}{\Delta h}, \qquad (21)$$

where Δv – the difference in particle velocities in layers, m/s; Δh – the distance between layers, m.

To describe the flow of the mixture in the drum, the Navier–Stokes equation is used, taking into account rheological properties:

$$\rho\left[\frac{\partial \vec{v}}{\partial t} + (\vec{v}\cdot\nabla)\vec{v}\right] = -\nabla p + \nabla\cdot\tau + \rho\vec{g}, \qquad (22)$$

where p – the pressure, N/m²; τ – the viscous stress tensor, N/m².

The mobility of the mixture determines its ability to move under the influence of external forces. To assess mobility, the following criteria can be applied:

$$M = \frac{1}{\tau_0 + \mu\dot{\gamma}}, \qquad (23)$$

where M – the mobility indicator; τ_0 – the yield strength, N/m².

The general formula of the mathematical model describes the dynamics of a particle in the drum, considering its trajectory, speed, and the influence of all external forces. In this case, the kinematic analysis must account for the gravitational and centrifugal components and the effects of friction and particle interactions. Since the mixing process is multifactorial, using Lagrange equations allows us to mathematically formalize it and obtain quantitative characteristics of particle motion in the mixer.

Thus, the following equation is used to describe particle behavior in the mixing space fully:

$$m\frac{d^2\vec{r}}{dt^2} = \vec{F}_{grav} + \vec{F}_{centr} + \vec{F}_{osc} + \vec{F}_{fr} + \nabla p + D\nabla^2 c, \quad (24)$$

Equation (24) describes the general dynamics of particle motion in the mixing space, considering all external forces acting on the system. By substituting the components of force effects into this equation, we obtain an extended form that more thoroughly reflects the kinematic and dynamic parameters of the mixing process:

$$m\frac{d^2\vec{r}}{dt^2} = -mg\vec{e}_z + m\omega^2 R\cos(\theta)\,\vec{e}_r - \mu N\vec{v}_{relative} + \\ +mR\frac{\partial^2 \sin(\theta(t))}{\partial t^2}\vec{e}_x + mR\frac{\partial^2 \cos(\theta(t))}{\partial t^2}\vec{e}_y + \nabla p + D\nabla^2 c, \quad (25)$$

where $\vec{e}_x$, $\vec{e}_y$, $\vec{e}_z$ – the unit vectors in the x-, y-, and z-directions.

Analysis of the obtained data allows us to estimate the distribution of active and passive zones within the mixing drum. It was found that intensive mixing occurs in zones located near the blades, while in the central and upper parts of the drum, areas with reduced particle velocity form (Figure 5).
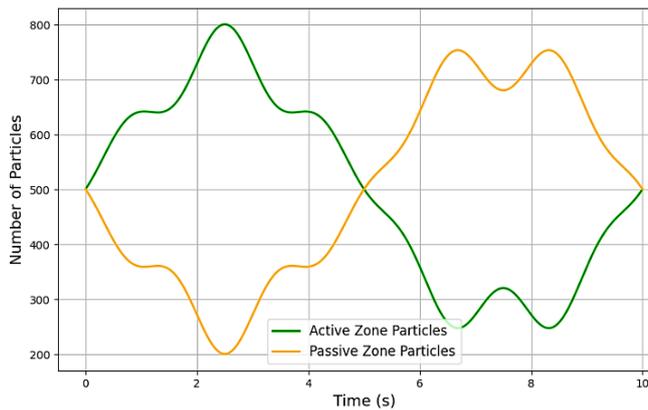
Figure 5 – Distribution of particles between active
and passive phases

This can cause local inhomogeneities in the mixture and decrease mixing efficiency. Using oscillatory motion allows us to significantly reduce the size of passive zones and ensure a more uniform particle distribution throughout the drum volume.

Their values were calculated at selected time points to analyze individual force components affecting particle motion quantitatively. The obtained data are summarized in Table 1, which serves as the basis for constructing graphical dependencies.

Table 1 – Calculated data for graphical representation

Time, s	F_{fr}, N	F_{ocs}, N	F_{centr}, N	F_g, N	F_{total}*, N
1	27.3	17.8	15.7	1.0	61.8
2	11.3	22.1	14.9	1.0	49.3
3	7.5	23.1	13.6	1.0	45.2
4	26.4	20.3	18.0	1.0	65.7
6	15.7	15.2	15.7	1.0	47.6
7	5.0	10.3	17.1	1.0	33.5
8	23.9	8.2	13.7	1.0	33.1
9	20.1	10.0	14.7	1.0	45.8
10	4.1	14.7	16.9	1.0	36.8

* F_{total} – the total force, N

The dynamics of particle motion within the mixing space are governed by several factors, with particle mass, drum geometry, and oscillatory motion characteristics playing the primary roles. Equation (24) provides a generalized framework for assessing their influence and determining the particle trajectories in the medium.

Oscillatory parameters play a particularly significant role in shaping the dynamics. Oscillation amplitude and frequency generate periodic flow disturbances that reduce passive zones and promote a more uniform particle distribution inside the drum. Their interaction with centrifugal and gravitational forces further defines the intensity of the mixing process. The analysis of individual components (Figure 6) demonstrates that none of the considered forces acts in isolation.

Their combined effect governs the particle motion inside the drum. The interplay of centrifugal, oscillatory, gravitational, and frictional forces produces a variable load whose magnitude changes over time.
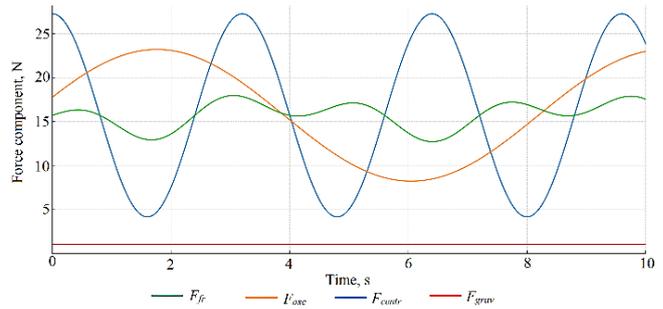

Figure 6 – Time-dependent variation of force components
in the mixing drum

To provide a more precise representation of the integral impact of these factors, it is appropriate to consider the resultant force, whose graphical dependence is shown in Figure 7.
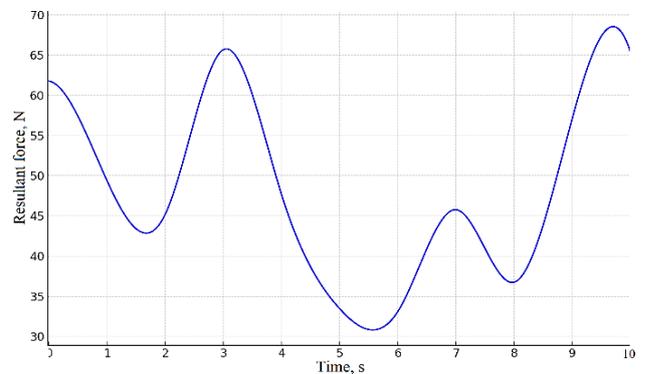

Figure 7 – Time-dependent variation
of the resultant force in the mixing drum

This dependence reflects the cumulative action of all force components and enables a comprehensive assessment of the overall mixing process.

Based on the graphical dependencies and the analysis of the obtained results, the optimal operating parameters of the mixing drum were identified. The generalized recommended values are summarized in Table 2.

Table 2 – Recommended operating parameters

Parameter	Value
Mass m, kg	1.5
Radius R, m	0.5
Rotation frequency n, rpm	40
Oscillation amplitude angle A, °	16
Oscillation frequency n_1, min^{-1}	14
Sliding friction coefficient f	0.3

Energy consumption is determined by the total work done to overcome gravity, friction, and inertia forces and ensure the drum's oscillatory motion. Optimizing energy consumption allows achieving high mixing efficiency with minimal energy use.

The total energy consumption in the mixing process depends on the combined effect of various physical factors, including gravity, friction, inertial effects, and the oscillatory motion of the drum. Each factor contributes to

energy consumption by changing the kinematics of particles in the mixing space. The total energy consumption of the mixing process can be divided into several components:

$$E_{tot} = E_{grav} + E_{fr} + E_{inert} + E_{osc}, \qquad (26)$$

where E_{grav} – the energy expended to overcome gravity, J; E_{fr} – the energy to overcome friction, J; E_{inert} – the energy of inertial forces, J; E_{osc} – the energy of oscillatory motion, J.

The energy of oscillatory motion E_{osc} is the energy required to provide periodic oscillations of the drum. The operation of the oscillating motion is described in terms of a change in the angle of inclination $\theta(t)$:

$$E_{osc} = \int_0^T M_{osc} \frac{d\theta(t)}{dt} dt, \qquad (27)$$

where M_{osc} – the moment of force providing oscillation, N·m.

The energy consumption to overcome gravity is determined by the mixture's mass and the height of its movement under the action of the drum's rotational motion. As the angle of inclination or the rotation speed of the drum increases, these costs may rise due to the need to lift the material to a greater height. However, excessive increases in these parameters may lead to a loss in mixing efficiency due to the stratification of mixture components.

Friction between particles and the drum walls is also an essential factor in energy costs. As friction increases, the energy required to move the particles rises, but it can also contribute to better mixing as particles stay in the active zone longer. Optimizing the friction coefficient allows finding a balance between minimizing energy costs and ensuring high-quality mixing.

Additional energy consumption is associated with the drum's oscillatory motion. These costs are determined by the frequency and amplitude of oscillations, as each cycle of changing the inclination angle requires additional mechanical work. At the same time, introducing oscillations helps reduce passive mixing zones, improving the final homogeneity of the concrete mix.

Figure 8 shows a graphical representation of the dependence between total energy consumption and mixing parameters. It can be seen that energy consumption is nonlinear, reaching its optimum value at a particular combination of drum rotation frequency, tilt angle, and oscillatory motion parameters. Further parameter increases can lead to excessive energy consumption without significantly improving mixing efficiency.

Within the framework of this study, an approach to energy optimization is adopted, which is based on the analysis of the dependence of E_{tot} on the parameters of the drum, in particular, the frequency of oscillations ω_{osc}, the angle of inclination θ_0, the radius of the drum R, and the parameters of the blades.

The analysis of these dependencies demonstrates that introducing oscillatory motion ensures a significantly more intensive growth of the homogeneity coefficient.
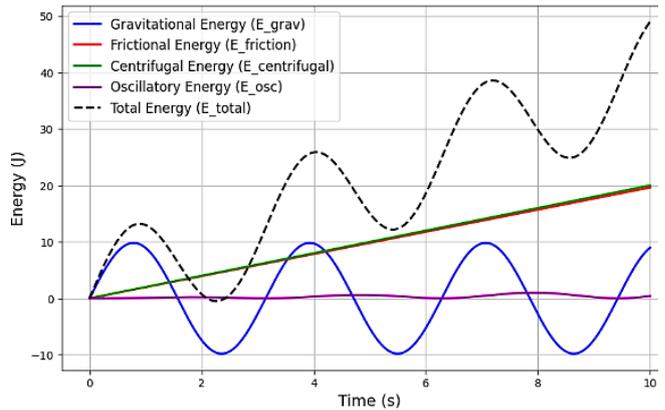

Figure 8 – Energy consumption schedule

The reduction of the ether is predicted by choosing the optimal materials of the blades that reduce friction, and the E_{osc} – through the use of more efficient oscillation generation mechanisms.

The time-dependent variation of the homogeneity coefficient κ, defined by equation (10), was analyzed for the quantitative evaluation of mixing efficiency. The coefficient κ was used to assess mixing quality, with its temporal evolution presented in Figure 9.
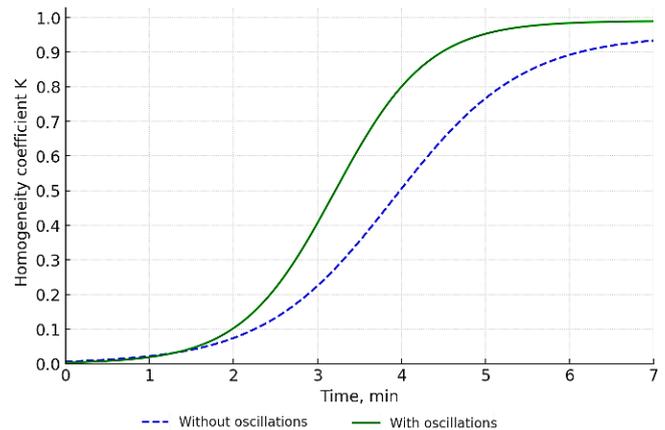

Figure 9 – Time-dependent variation of the homogeneity coefficient with and without oscillations

A noticeable increase in κ is observed as early as 2 min, reaching $\kappa = 0.8$ by 4 min, corresponding to high-quality mixing. In contrast, homogenization proceeds more slowly without oscillations: rapid growth begins only after 5 min, and satisfactory value $\kappa = 0.9$ is reached only at 6–7 min.

Thus, applying oscillatory motion reduces the time required to achieve high mixing quality by nearly half and ensures greater stability of homogeneity indicators.

Thus, oscillatory motion substantially accelerates homogenization and reduces the time required to achieve high-quality mixing.

This confirms that applying oscillations in a concrete mixer is an effective approach to enhancing process intensity and ensuring stable properties of the final mixture.

5 Discussion

The study results showed that concrete mixing in a gravity concrete mixer is a complex, multifactorial phenomenon dependent on the kinematic motion of particles, the drum's design features, and the system's dynamic characteristics. The data confirmed that the main factors affecting mixing efficiency are the drum rotation frequency, inclination angle, and the blade location. The optimal combination of these parameters ensures a uniform distribution of mixture components and reduces the formation of passive zones, which is crucial for improving the final product's quality.

Analysis of active and passive zones inside the concrete mixer drum showed that intensive mixing occurs near the blades, while in the central and upper parts of the drum, areas with low particle velocity often form. These zones contribute to inhomogeneities in the concrete mixture, which can negatively affect its mechanical properties after hardening. Additional design solutions, such as oscillatory drum movement, can significantly reduce the impact of passive zones and increase mixing efficiency.

Graphical dependences confirmed that the interaction of gravitational, centrifugal, and friction forces determines particle movement in the drum. It was found that particles may not reach the required mixing level at insufficient rotation speed, leading to increased heterogeneity in the mixture composition. On the other hand, excessive drum rotation speed can cause stratification, where particles, under centrifugal forces, are pressed against the drum walls and do not actively participate in the mixing process. The optimal speed should balance particle movement and their interaction with the drum blades.

The results of mathematical modeling confirmed the effectiveness of using the drum's oscillatory motion to improve mixing efficiency. Adding a harmonic component to the drum motion significantly increased the active mixing area, reduced passive zones, and ensured mixture homogeneity. These results align with previous studies in this field, indicating the potential for improving mixing efficiency by periodically changing the drum's inclination angle.

The study confirmed that the concrete mix's viscous properties significantly affect the mixing process kinematics. At high viscosity, particle movement speed decreases, which can lead to material accumulation in certain drum areas. This, in turn, worsens mixing quality and may require a longer process or an increase in drum rotation speed.

The analysis of the obtained data confirmed the validity of the selected design solutions to improve mixing efficiency. Using kinematic modeling allowed tracking the nature of particle movement in the mixing space and assessing the influence of the drum's oscillatory motion on the uniformity of mixture component distribution. The obtained results can serve as the basis for further improvement of the concrete mixer design and optimization of mixing technology.

The research confirmed the effectiveness of new approaches to optimizing the concrete mixing process.

However, some issues remain that require further study. In particular, the influence of different types of concrete mixtures on the kinematics of the mixing process, as well as the possibility of using alternative drum rotation modes. Further research in this area can focus on experimentally studying different variants of the drum's oscillatory motion and developing new mathematical models for even more accurate predictions of concrete mixture behavior.

Thus, the analysis proves that introducing new methods for regulating the kinematics of mixing significantly improves the quality of concrete mixtures, reduces energy consumption, and enhances the technological characteristics of mixing equipment. The results create the prerequisites for further improvement in concrete mixture production technologies and their widespread implementation in industrial construction.

6 Conclusions

An analysis of the concrete mixing process in a gravity mixer was conducted. As a result, a mathematical model describing the particle motion kinematics inside the mixing drum was developed. The proposed model accounts for the influence of gravitational force, centrifugal acceleration, friction, and drum oscillatory motion, enabling a more accurate determination of particle trajectories and their spatial distribution within the working volume of the mixer. Motion equations and hydrodynamic and rheological models made it possible to identify the key parameters affecting mixing efficiency.

It was established that drum rotation speed, tilt angle, blade geometry, and oscillatory motion significantly influence the mixing process. The study demonstrated that an optimal drum speed of 40 rpm minimizes passive zones and ensures uniform component distribution. When the rotation speed drops below 20 rpm, the share of passive zones in the working volume may exceed 20–25 %, leading to uneven mixing and a 10–15 % reduction in the mixing intensification coefficient.

The proposed oscillatory motion model assumes a periodic variation of the drum tilt angle relative to the horizontal by $\pm 16°$ at a 12–14 min^{-1} frequency. This regime redistributes particles between passive and active zones, increasing the share of the active zone from 72 % to 85 % and reducing the coefficient of variation of component concentration from 0.21 to 0.15. Mathematical simulations confirmed that introducing oscillatory motion within this parameter range increases the mixing intensification coefficient κ by 15–18 % compared with non-oscillatory mixing.

The results indicate that combining the optimal rotation speed of 40 rpm with oscillatory motion of 16° amplitude and 14 min^{-1} significantly intensifies the mixing process, reduces the passive-zone area to minimal levels, and improves mixture uniformity by 12–15 % in terms of the coefficient of variation.

The proposed approach also enhances energy efficiency by improving mixing quality without substantially increasing energy consumption.

The determined mixing intensification coefficient provides a quantitative measure of the effectiveness of altering particle motion kinematics under oscillatory conditions and may be applied to optimize further the design and operating regimes of concrete mixers in industrial practice.

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