Mathematical Model of Single-Channel Infocommunication Node with Several Packet Flows

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Abstract **— Despite some shortcomings of a mathematical apparatus of queuing theory, it nevertheless remained as a simple technology of the solution of problems of infocommunication nodes modeling. In this work it will be reviewed an example of the solution of such task for single-channel network node in case, that on his entrance incomes the superposition of equal priority streams of packages with various intensity. In this work, we obtain relations for calculating the distribution of states of a Markov system in the stationary condition. Possessing information on the distribution of stationary states, it is possible to calculate the quality of service parameters in the nodes of telecommunication networks: the average delay in the node, the average queue length, and the average number of packet losses. Quality of service parameters can be calculated for each of the streams entering the superposition, and for the composite flow.**

Keywords — Networks, mathematical modeling, Queueing system, state diagram, quolity of service.

The Queuing Systems (QS) are the most widespread mathematical tool applied for the analysis of processes in infocommunication networks. Classical works in the field of the theory of QS are [1-2]. Work [2] generalize the theory of QS, considering objects of data domain as the set of "queues" and "channels of service" with arrived packets on service to their input. In these works single-channel and multi-channel QS are considered with a normal, uniform and exponential packets flows. The general principles of delay period calculation of the request, output, queues are justified depending on the QS type and a packet flow. In work [1] considered the theory of QS for the reasons of structure and network topology of ARPONET (a prototype of future INTERNET). In this work the main assumptions which allowed to receive analytical dependences for assessment of network quality are justified. "Kleinrock-Jackson's model" represents a network as a set of independent networks, where each one represents single-channel QS with an unlimited infinite power of a packets source. The "Approximation by Kleinrock's independence" is an assumption about independence of a flow in each node, independence of packet arrival time and its length from the previous packets, and at the same time it is considered that the packets flow is stationary and arrival time and service of packets are distributed by the exponential law. In the work [1] was formulated the key indicators of quality of an infocommunication network:

- average time of packets service;
- effective throughput in case of restrictions on service failure probability, network topology, network cost (directly proportional length and the maximum network transmission capacity);
- service failure probability.

I. ANALISYS OF LITERATURE.

Restrictions and assumptions of work [1] afterwards will form the basis of a set of researches of quality assessment of an infocommunication network. Further works are set as the purpose simulation on the basis of QS of a network of different types. The ALOHA models for satellite channels [3], models of multistation access for networks of an air radio communication [4], wired models (including fiber-optic) "point-to-point" networks were reasonable [3,5]. Based on QS it was constructed two-level models, including models of the backbone network, which servicing a traffic between complexes of the nodes and terminal networks, which describes terminal access to complexes of nodes [3, 6-8]. However, the redistribution of a traffic, appearance of new technologies of data transfer [9-11], formation of wide area networks related to growing of information support in the 80-90th years, lead to necessity of formation of new adequate models of infocommunication networks [12]. For the purpose of adequate process descriptions in networks begin the wide usage models with the restrictions on buffer gears [13], models of the adaptive traffic distribution [14], model with priority dispatching of the incoming packets, models of multi-channel service [15]. However, all mentioned models were the advanced models of Kleinrock-Jackson and had assumptions of "approximation by Kleinrock's independence". Finally, there was a question of necessity of models enhancement beyond the scope of the theory of QS as these models do not describe with necessary adequacy the all range of processes

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in modern infocommunication networks, what is fairly marked in work [1]. Defects of QS were excluded by models based on other mathematical tools.

Removal of assumption of flow stationarity in infocommunication networks and absence of influence of jitter was solved by models based on the theory of indistinct sets [4, 16] for networks with a high level of traffic intensity fluctuation. Impossibility of receiving analytical relations for characteristics of QS in the conditions of an ultimate load (for models of networks based on QS assumption that loading of a network tends to zero) solved by creation of models with useage of a mathematical tool of the tensor analysis. Besides, the tensor analysis along with the theory of fractals and self-similarity is applied to obtain dependences of traffic distribution on structure and network topology and also approximation of package loading of an infocommunication network. Removal of assumption about independence traffic income in each node of a network was solved by models based on the graphs which use a mathematical tool of Petri nets. The last, lead to necessary of decision of the problem of optimum routing which was solved with usage of a mathematical tool of graphs, based on games theories, theories of decision-making, the theory of a fuzzy logic.

Thus, despite some shortcomings of a mathematical tool of QS, it nevertheless remains simple technology of the solution of problems of modeling of nodes of infocommunication networks. We will review in this work an example of the solution of such task for single-channel node of network in case on his entrance incomes the superposition of package streams with equal priority but with various intensity.

II. Statement of the research problem

Let's consider the technology of the analysis of infocommunication network node, using Markov models of their functioning.

Let us incomes the superposition of two Poisson streams with intensity λ_1 and λ_2 on node entrance. We will assume that flows intensity of service requests equal to μ_1 and μ_2. We will consider further that lengths of flows queues requests of the first and second types are n_1 and n_2. We enter a set of possible statuses of a node. Let E_{ik} – the node status corresponding to a situation when in system there are i requests of the first type and k requests of the second type. The state diagram of such system is given in a figure 1.

III. Decomposition of a difficult task.

For the task solving we use technology of phase enlargement of statuses. Lets' suppose that a set of statuses $\{E_{i0}, E_{i1}, E_{i2}, ..., E_{in_2}\}$ belong to a layer i. Let's enter the integrated status $E_{\Gamma i} = \{E_{i0}, E_{i1}, E_{i2}, ..., E_{in_2}\}$. Then in terms of the integrated statuses the state diagram reduces to the following form (fig. 2).

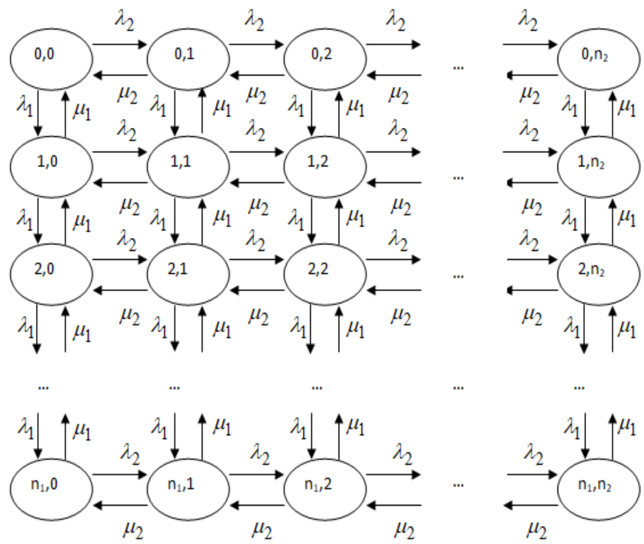

Fig. 1. State diagram of the system

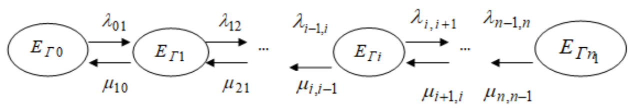

Fig. 2. State diagram of the integrated system

Let's calculate intensity of transitions for a state diagram with integrated statuses. Intensity $\lambda_{i,i+1}$ of transition from $E_{\Gamma i}$ to $E_{\Gamma,i+1}$ is equal to:

$$\lambda_{i,i+1} = \sum_{k=0}^{n_2} \lambda_1 \cdot \hat{P}_{ik} = \lambda_1 \sum_{k=0}^{n_2} \hat{P}_{ik}, \quad i = 0,1,...,n_1 - 1,$$

where

$\hat{P}_{ik}$ is conditional probability that system staying in the $k-th$ state of the $i-th$ layer, on condition it is in this layer.

$$\sum_{k=0}^{n_2} \hat{P}_{ik} = 1 \text{, that } \lambda_{i,i+1} = \lambda_1, \quad i = 0,1,..., n_1 - 1.$$

In the same way let's find intensity of transition from a status $E_{\Gamma,i+1}$ в $E_{\Gamma i}$. We have:

$$\mu_{i+1,i} = \sum_{k=0}^{n_2} \mu_1 \cdot \hat{P}_{i+1,k} = \mu_1 \sum_{k=0}^{n_2} \hat{P}_{i+1,k} = \mu_1, \quad i = 0,1,...,n_1 - 1.$$

IV. Solution of subtasks superposition.

Let's find distribution of probabilities that system stays in each of the layers corresponding to the integrated statuses now. Taking into account the received ratios for intensity of transitions the state diagram takes the form provided in a figure 3.

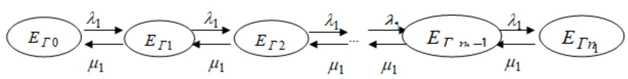

Fig. 3. State diagram of the integrated statuses

Let's write the appropriate system of equations for probabilities of the integrated statuses.

$$\begin{cases} \mu_1 P_1 - \lambda_1 P_0 = 0, \\ \lambda_1 P_0 + \mu_1 P_2 - (\lambda_1 + \mu_1) P_1 = 0, \\ \dots\dots\dots\dots \\ \lambda_1 P_{i-1} + \mu_1 P_{i+1} - (\lambda_1 + \mu_1) P_i = 0, \\ \dots\dots\dots\dots \\ \lambda_1 P_{n_1 - 1} - \mu_1 P_{n_1} = 0. \end{cases} \quad (1)$$

Let's input $z_i = \lambda_1 P_{i-1} - \mu_1 P_i$ [18]. Then the system of equations (1) will take a form:

$$\begin{cases} z_1 = 0, \\ z_1 - z_2 = 0, \\ \dots\dots \\ z_i - z_{i+1} = 0, \\ z_{n_1} = 0. \end{cases}$$

From this $z_1 = z_2 = \ldots = z_{n_1} = 0$, and

$$\lambda_1 P_{i-1} + \mu_1 P_i = 0, \quad i = 1,2,\ldots, n_1 .$$

Wherein

$$P_i = \frac{\lambda_1}{\mu_1} P_{i-1}, \quad i = 1,2,\ldots, n_1 .$$

Then

$$P_1 = \frac{\lambda_1}{\mu_1} P_0,$$

$$P_2 = \frac{\lambda_1}{\mu_1} P_1 = \left(\frac{\lambda_1}{\mu_1}\right)^2 P_0,$$

$$\dots\dots\dots\dots\dots\dots$$

$$P_i = \left(\frac{\lambda_1}{\mu_1}\right)^i P_0,$$

$$\dots\dots\dots\dots\dots\dots$$

$$P_n = \left(\frac{\lambda_1}{\mu_1}\right)^n P_0.$$

The value P_0 is found from a normalization condition

$$\sum_{i=0}^{n} P_i = P_0 \sum_{i=0}^{n_1} \left(\frac{\lambda_1}{\mu_1}\right)^i = 1 .$$

From this it follows that $P_0 = \dfrac{1}{\sum_{i=0}^{n_1} \left(\frac{\lambda_1}{\mu_1}\right)^i}$.

Then

$$P_i = \frac{\left(\frac{\lambda_1}{\mu_1}\right)^i}{\sum_{l=0}^{n_1} \left(\frac{\lambda_1}{\mu_1}\right)^l}, \quad i = 1,2,\ldots, n_1 .$$

Let's input $\rho_1 = \dfrac{\lambda_1}{\mu_1}$. Wherein

$$P_i = \frac{\rho_1^i}{\sum_{l=0}^{n_1} \rho_1^l}, i = 1,2,\ldots, n_1 . \quad (2)$$

As $\rho_1 < 1$, the sequence of $1, \rho_1, \rho_1^2, \ldots, \rho_1^{n_1}$ forms the decreasing geometrical progression with the first member, equals 1, and a denominator ρ.

That's why:

$$\sum_{l=0}^{n_1} \rho_1^l = \frac{1 - \rho_1^{n_1 + 1}}{1 - \rho_1} .$$

Then distribution of probabilities of the integrated statuses has an appearance:

$$P_i = \frac{\rho_1^i (1 - \rho_1)}{1 - \rho_1^{n_1 + 1}}, \; i = 0,1,2,\ldots, n_1 . \quad (3)$$

V. THE SOLUTION OF THE TASK.

We will find the conditional distributions of probabilities of statuses of system for each of layers now. These distributions, evidently, are identical as the state diagram for each of them has the appearance shown in fig. 4

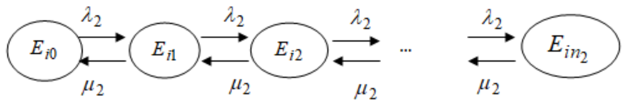

Fig. 4. State diagram of the $i - th$ layer.

The technology of the analysis of this state diagram is absolutely similar to described one above. The conditional probabilities are equal:

$$P_{i,k} = \frac{\rho_2^k (1 - \rho_2)}{1 - \rho_2^{n_2 + 1}}, \; k = 0,1,2,\ldots, n_2 , \quad (4)$$

where $\rho_2 = \dfrac{\lambda_2}{\mu_2}$.

Then absolute probability of stay of system in i-th status k-th layer is defined by relation:

$$P_{i,k} = P_{\Gamma k} \cdot P_i = \frac{\rho_1^i \cdot \rho_2^k (1 - \rho_1)(1 - \rho_2)}{(1 - \rho_1^{n_1 + 1})(1 - \rho_2^{n_2 + 1})},$$

$$i = 0,1,\ldots, n_1 ; \quad k = 0,1,\ldots, n_2 . \quad (5)$$

Estimation of characteristics of QoS in a network node.

If the queue length for requests of both types is not restricted, then the relation (5) becomes simpler

$$P_{i,k} = \rho_1^i \cdot \rho_2^k (1 - \rho_1)(1 - \rho_2). \quad (6)$$

VI. IMPLEMENTATION

Let's calculate the average values $\overline{n}_1, \overline{n}_2$, the number of applications of the first and second type, standing in the queue. It's clear that

$$\overline{n}_1 = \sum_{i=1}^{\infty} i \sum_{k=0}^{\infty} P_{i,k} = \sum_{i=1}^{\infty} i \sum_{k=0}^{\infty} \rho_1^i \rho_2^k (1 - \rho_1)(1 - \rho_2) =$$

$$= (1 - \rho_1)(1 - \rho_2) \sum_{i=1}^{\infty} i \rho_1^i \sum_{k=0}^{\infty} \rho_2^k = (1 - \rho_1) \sum_{i=1}^{\infty} i \rho_1^i =$$

$$= (1-\rho_1)\rho_1 \sum_{i=1}^{\infty} i\rho_1^{i-1} =$$

$$= (1-\rho_1)\rho_1 \sum_{i=1}^{\infty} \frac{d(\rho_1^i)}{d\rho_1} = (1-\rho_1)\rho_1 \frac{d}{d\rho_1}\left(\sum_{i=1}^{\infty} \rho_1^i\right) =$$

$$= (1-\rho_1)\rho_1 \frac{d}{d\rho_1}\left(\frac{1}{1-\rho_1}\right) = \frac{(1-\rho_1)\rho_1}{(1-\rho_1)^2} = \frac{\rho_1}{1-\rho_1}. \quad (7)$$

Similarly to

$$\overline{n}_2 = \sum_{i=0}^{\infty}\sum_{k=1}^{\infty} kP_{i,k} = \sum_{i=0}^{\infty}\sum_{k=1}^{\infty} k\rho_1^i \rho_2^k (1-\rho_1)(1-\rho_2) =$$

$$= (1-\rho_2)\sum_{k=1}^{\infty} k\rho_2^k = \frac{\rho_2}{1-\rho_2}. \quad (8)$$

Now it is easy to calculate the average duration of waiting of package in queue to service. Let the next request come to system at the moment when in queue there are i packages of the first type and k packages of the second one. At the same time the average time of waiting of the beginning of service for the income packages is equal to $\overline{\tau}_{ik} = \frac{i}{\mu_1} + \frac{k}{\mu_2}$.

Averaging this relation by all i and k

$$\overline{\tau} = \sum_{i=0}^{\infty}\sum_{k=0}^{\infty}\left(\frac{i}{\mu_1} + \frac{k}{\mu_2}\right)P_{i,k} =$$

$$= \sum_{i=1}^{\infty} \frac{i}{\mu_1} \sum_{k=0}^{\infty} P_{i,k} + \sum_{k=1}^{\infty} \frac{k}{\mu_2} \sum_{i=0}^{\infty} P_{i,k} = \frac{\overline{n}_1}{\mu_1} + \frac{\overline{n}_2}{\mu_2}. \quad (9)$$

The received relations easily are generalized on a case when the flow of packages represents superposition of m flows with income intensity respectively is equal to $\lambda_1, \lambda_2, ..., \lambda_m$ and intensity of service - $\mu_1, \mu_2, ..., \mu_m$. At the same time the median number of requests of j-th type, being in queue is equal

$$\overline{n}_j = \frac{\rho_j}{1-\rho_j}, \quad j = 1,2,...,m, \quad (10)$$

and the average duration of waiting of the beginning of service for the package which came to system in arbitrary timepoint is defined by a relation

$$\overline{\tau} = \sum_{j=1}^{m} \frac{\overline{n}_j}{\mu_j} = \sum_{j=1}^{m} \overline{n}_j \cdot \overline{\theta}_j, \quad (11)$$

where $\overline{\theta}_j = \frac{1}{\mu_j}$ is average time of service requests of j-th type.

VII. Conclusions

Meanwhile, despite a row of essential shortcomings of models on the basis of the theory of QS is given above they continue to remain the most widespread tools of the analysis of infocommunication networks. It is caused by deep study of a method of application of this mathematical tool, a large number of models for different objects as a part of an infocommunication network (a computer, TCS, terminals, switching nodes) and also the fact that, rather wide class of tasks can be formalized by this mathematical tool.

References

[1] L. Kleinrock, *Vichislitelnie seti s ocherediami* [*Queuing networks*]. Moscow, Russia: Mir, 1979. (In Russian).

[2] A. Koffman. *Massovoe obslujivanie* [*Queuing theory*]. Moscow, Russia: Mir, 1965. (In Russian).

[3] O.I. Aven. *Ocenka kachestva I optimizacii vichislitelnih system* [*Quality estimation and optimization in systems*]. Moscow, Russia: Nauka, 1982. (In Russian).

[4] P.A. Budko. *Upravlenie v setiah sviazi* [*Data networks managing*]. Moscow, Russia: FizMatLit, 2003. (In Russian).

[5] S. Porozki *Modelirovanie algoritma marshrutizacii transportnoi ATM seti* [*Route algorithm modeling in transport ATM network*] Moscow, Russia: Elekrosviaz, №10, 2000 .- pp. 16-19. (In Russian).

[6] V. Olifer. *Komputernie seti. Principi, tehnologii, protokoli* [*Computer networks. Principles, technologies, protocols*]. St Peterburg, Russia: Piter, 2010. (In Russian).

[7] V.A Yakubaitis.. Informacionno vichislitelnie seti [*Information networks*]. Moscow, Russia: Finansi I statistika, 1984. (In Russian).

[8] P.A. Budko. Vibor propusknih sposobnostej kanalov pri sinteze settej [Choosing of income probability of channels during network sintesing] in *Phisics of wave systems* - T.3. - №3-4, 2000. – pp.68-72. (In Russian).

[9] E. Tanenbaum. *Komuternie seti* [*Comuter neworks*]. St Peterburg, Russia: Piter, 2010. (In Russian).

[10] L.L. Peterson, Larry L. Peterson, and Bruce S. Davie. *Computer Networks, Fifth Edition: A Systems Approach,* Burlington:Elsevier, 2012. – 884p.

[11] Ruhe G and G. Ruhe, *Algorithmic Aspects of Flows in Networks,* Kluwer, 1991. - 212p.

[12] B. Stallings. *Sovremennie komputernie seti* [*Modern computer networks*]. St Peterburg, Russia: Piter, 2003. (In Russian).

[13] Pustovoitov P. *Flow management in routers by criteria of average queue length*, “Modern problems of radio engineering, telecommunications and computer science”: *Proceedings of the XIth International Conference TCSET’2012* Lviv – Slavske, Ukraine, February 21–24, 2012. – P.363.

[14] Pavel Pustovoitov. *Routing technology in networks taking into account traffic dynamics*, Proceedings of “System Analysis and Information Technologies”: Materials of the 14th International Scientific and Technical Conference NTUU KPI,, 2012, pp.116-117.

[15] Pavel Pustovoitov. *Managing the discipline of queues in routers by the criterion of the maximum queue length*, materials of “Modern problems of radio engineering and telecommunications: materials of the 8th International Youth Scientific and Technical Conference”, Sevastopol, SevNTU, 2012.

[16] Pavel Pustovoitov. *Managing queues in routers by the criterion of the average number of queues, whose length is not below critical* , Proceeding of “System analysis. Computer Science. Management (SAIU-2012)”: materials of the III International Scientific and Practical Conference (Zaporozhye, March 14-16, 2012), CPU. - Zaporozhye: CPU, 2012. – C.243-244.